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Imperatives of Extinguishment: *Kartinyeri v The Commonwealth of Australia*

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Abstract. Via an analysis of the reasons for judgement in *Kartinyeri v The Commonwealth of Australia* this paper contends that an imperative of extinguishment of Aboriginal sovereignty continuously informs Australian law. With attention to the different, albeit interactive, practices of justification and legitimation in the reasons given, the imperative mood or modality of a doctrinal assumption of extinguishment that is made in the ruling system of Australian law is located in the official voice or internal point of view of that law.

1. Introduction: Toward a Concept of the Wrong of Law

This paper is a case study focused on the reasons for judgement given by the Australian High Court in *Kartinyeri v The Commonwealth of Australia*.[1] It is part of a broader endeavour to conceptualise what for some years I have called 'the wrong of law'. One of the difficulties of this endeavour is to remove the sense of 'wrong' in that phrase from the tyranny of moral points of view. Part and parcel of the justificatory dimension of jurisprudential and philosophical discourse on modern law, that tyranny is deeply embedded in narrative and conceptual thought about law. As much as the form of modern law differs from that of the law of Athens to which Plato directed his *Crito* and as much as changing forms of law bring with them new justificatory techniques and legitimative strategies, a moral obligation to obey, respect or even love the law that is in force in a political community is continuously reaffirmed.

[1] (1998) 152 ALR 540. I would like to thank participants in the workshop 'Law Violence and Colonialism II' held at the Altonaer Stiftung für philosophische Grundlagenforschung on 9th–11th May, 2008 for a diversity of responses to another version of this study. Those responses have continuously informed this re-writing.

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Kartinyeri decided that racially discriminatory legislation was constitutionally valid. To bring the difficulty to a point I shall step right into it by characterising the decision as a legal expression of the racism in Australian society. The characterisation takes a short way with a troublesome term, 'racism' and with the issue of beneficial and detrimental departures from law's norm of formal equality. At least in the context of social relations between Aboriginal and non-Aboriginal Australians the 'race' part of 'racism' is clearly enough marked. It is the 'ism', working its indiscriminate bundling of uncountable and diverse phenomena — acts, attitudes, events, structures — and, attached to race, packing moral value, that is troublesome. To write or talk of 'indiscriminate bundling' suggests a need to discriminate in concept formation. If that theoretical necessity, confronted by modern law's norm of formal equality, is pursued by distinguishing beneficial and detrimental departures from the norm, the moral force of 'racism' is only strengthened. It might be too much to say that moral values repel theoretical intentions, but they take them hostage. The characterisation and the phrase 'legal expression of racism' is meant to raise this as a political problem which requires theoretical address.

The address is made to readers who locate themselves on the political left and I place my work in a general genre of critical legal theory. It differs from most other work being done in that genre in that, taking up Hegel's idea of replacing the old metaphysics by a formal, dialectical logic, it supposes thought's logical foundation.[2] Further, against Hegel it is my persuasion that the method and logic appropriate to this task is mathematical not philosophical and this opens a further difference regarding the disciplines that are included in an interdisciplinary legal theory.[3] The motivating point here goes to the difficulty of the previous paragraph. How is

[2] [Kerruish and Petersen 2006].

[3] It would suit me well if, at this foundational level, I could invoke the work of a philosopher who is currently read by at least some legal theorists. Unfortunately that is not the case. Alain Badiou does indeed propose a mathematical theory, ZFC (Zermelo Fraenkel with the axiom of choice) set theory, in the place of ontology, but his enterprise is to give a philosophical interpretation to that theory. Rather than *replacing* metaphysics by a mathematical theory that seems to me to be a *reconstruction* of metaphysics, indeed on the basis of an axiom (extensionality) which is constitutive of set theory but is inconsistent with the dialectical higher order logic at the foundation of my approach ([Petersen 2007] at 128f). In these circumstances the best I can do is refer readers to J.N. Findlay's perception of Cantor's generation of transfinite numbers

the 'wrong' of the wrong of law to be thought free of the sense with which law imbues notions of 'wrong'? One might say morally or ethically but I don't see that these notions of 'wrong' have the independence required, even if certain 'ethical turns' do provide a distinctively distanced theoretical approach. Let me just say, given that this case study is part and parcel of finding an answer to that very question, that the sense aimed at is certainly *not* that which inhabits notions of 'theoretical' divorced from 'practical' reason; not then the sense of 'wrong' that is tied to mistake. Here at least is a point of rapprochement: the sense is tied to the claim that something is rotten at the foundations of legal thought.

That said, and said with intent to raise a foundational issue that bears on the 'wrong' of the wrong of law, this paper proceeds from an assumption regarding the form of modern law that is phenomenal rather than (formally) logical/conceptual. Modern law has its institutional embodiments and its social practices. It creates a world of its own, legal thought, and in and by so doing catches thought in the dilemma that Marx encountered in the chapter on economic value with which he began *Capital*.[4] This world of doctrines and their validity or of interpretive practices and their values is removed from what are lamely termed its 'material conditions', but even so, law's business is that of regulating, ruling and ordering social life. Historically, the secret of the common law's success has been to fashion its doctrines from habitual and customary practices and the relations of power and position within which they take place and return them as products of its genius for justice or good order: reasonableness classically; in contemporary theory, fairness.

One should give credit where it is due and this exchange is ingenious. That might serve as an apology for the pages of analysis of the reasons for judgement in one Australian case that follow. But I direct it here to one particular aspect of this ingenious exchange: the way in which the particularist case by case approach of judicial praxis works together with the universalising tendency of formalisation to doubly isolate the decision reached and justified.[5] The meaning of the litigation for the plaintiffs

and Gödel's incompleteness theorems as "excellent and beautiful examples of Hegelian dialectic" ([Findlay 1976] at 6f, cited in [Kerruish and Petersen 2006] at 78–9).

[4] [Kerruish 2007] for critical exegesis and analysis.

[5] That in its modern form, law involves formalisation and formalism and that these are practices which enable the shift from a logic immersed in particular cases to one

is replaced by the legal meaning of the dispute through formalisation.[6] The social and political context of the litigation, both as such contextual considerations may influence the outcome and as the decision may alter the context are excluded by attention to the particular case. I will fill out these general comments in their application to *Kartinyeri* in the following section. The general point here, going back to the ingenious exchange which is my topic, is simply the effectiveness of this praxis as the stuff and matter of social life is spun into the gold of doctrine; into a kind of universal equivalent of persons who as bearers of rights and duties are at once equal and unequal.

2. Precis

> The fiction by which the rights and interests of indigenous inhabitants in land were treated as nonexistent was justified by a policy which has no place in the contemporary law of this country.[7]

The plaintiffs in *Kartinyeri*, Doreen Kartinyeri and Neville Gollan, sought a declaration from the High Court that an Act of the Commonwealth Parliament, the Hindmarsh Island Bridge Act 1997, was constitutionally invalid. The Act excluded specified places, Hindmarsh Island and an adjoining bank of the river in which it lies, from the scope of the Aboriginal and Torres Strait Islander Heritage Protection Act 1984. Its operative provisions withdrew powers to protect culturally significant Aboriginal sites from damage or destruction vested in the Commonwealth government by the 1984 Act in respect to these areas. Its effect was to withdraw from the plaintiffs and their community procedural rights for the protection of sites created by the 1984 Act.

Facts agreed in the pleadings were bare: that the plaintiffs were members of the Ngarrindjeri people who are of the Aboriginal race; that they had applied for and been granted, a declaration made by the responsible Minister under the Heritage Protection Act which protected the sites in question for a period of 25 years; that this declaration was invalidated on procedural grounds by the High Court in earlier proceedings and that

which as independent of them, aspires to universality is Bourdieu's astute sociological observation ([Bourdieu 1987] at 83).

[6] On this point, differently worked cf. [Barthes 1973a].

[7] *Mabo and Others v The State of Queensland* (No.2) (1992) 107 ALR 1 per Brennan J. at 28.

subsequently the Bridge Act withdrew from the Minister the authority to act in relation to these specific sites.

The question of law before the Court was whether the Bridge Act was invalid in falling outside any of the heads of Commonwealth legislative power specified in the Constitution.[8] It was agreed between the parties that the only relevant head of power was the 'race power' contained in s.51(xxvi). Originally (1901) formulated with an exclusion of Aboriginal peoples from its ambit, this placitum was amended following a Constitutional amendment in 1967. The text of the provision (as given in the cited copy of those judgements which include it) is:

> The Parliament shall, subject to the Constitution, have power to make laws for the peace, order and good government of the Commonwealth with respect to:
>
> ...
>
> (xxvi) The people of any race, ~~other than the aboriginal race in any State~~, for whom it is deemed necessary to make special laws.

Argument for the plaintiffs contended first, that any such laws must extend to all members of a given race, second that the section authorises only laws for the benefit of the people of a race or, in the alternative, for the benefit of the people of the Aboriginal race. The alternative within the second argument drew on the common understanding that the intention of the Commonwealth Parliament and the Australian electorate in framing and supporting the 1967 Constitutional amendment was benevolent: to alter the Constitution by ending the exclusion of Aboriginal peoples from the census[9] and to empower the Commonwealth Parliament to pass country wide laws furthering Aboriginal welfare.[10] The Human Rights and Equal Opportunity Commission as interveners in the case, pressed obligations of a legal character on members of the United Nations to protect human rights and argued for Constitutional construction

[8] Within the federal structure of State and Commonwealth governments established by the Australian Constitution the Commonwealth has only those legislative powers specified in the Constitution. These are plenary powers embodying parliamentary sovereignty within a federal system of representative democracy.

[9] Effected by a repeal of s.127 of the Constitution.

[10] The Heritage Protection Act is an example of such a law. In the reason of the law, until the passage of the Bridge Act the power had only been used to pass legislation intended to benefit Aboriginal and Torres Strait Islander peoples. The Native Title Act 1993 already makes that reason dubious in my view.

appropriate to such obligations. The plaintiffs argued more narrowly that the 1967 Constitution Amendment Act, if capable of a construction that would make it consistent with Australia's international legal obligations, should be so construed.

The Bridge Act was declared valid. If the question decided by *Kartinyeri* is whether the Australian Constitution authorises the Commonwealth to pass racially discriminatory legislation the answer, by five to one, is yes. But the grounds for and circumstances, beyond those of this case, in which it may do so remain undecided. On the question of the interpretation of the race power, the judgements present a three-way division. Two judges (Kirby and Gaudron) interpreted it as in effect confined to beneficial discrimination, two judges (Gummow and Hayne) thought it authorised both adverse and beneficial discrimination, and two judges (Brennan and McHugh) thought it improper to address that issue. In their view the case was not about the race power. It was about the nature of plenary legislative power, specifically, about the idea that what parliament may enact it may amend or repeal. In their opinion the nature of the power in the common law conception of it worked to prevent an issue on the meaning of the race power coming before the Court. It was therefore not only unnecessary but indeed mistaken to address the interpretation of the race power at all.

On this issue the judgements form different groups again: a three to three split. Gaudron agreed with Brennan and McHugh that the Bridge Act was valid due to the plenary character of Commonwealth legislative power, giving her opinion on the interpretation of the race power, *obiter*. Performatively, she is ambivalent on what the case is 'about'. She does not think it improper to address the issue, but joins Brennan and McHugh in taking the nature of plenary legislative power to dispose of this particular case. Gummow, Hayne and Kirby reject the view that the plenary character of the Commonwealth's legislative power disposed the case. For them the case is about the race power which must be interpreted in order to determine the validity of the Bridge Act.

The issues before the Court in *Kartinyeri* were consequent on a change of government at federal level (March, 1996) and the new government's passage of the Bridge Act. It brought to a bitter end a confrontation of many years duration. A small business couple wished to develop a marina and other facilities on Hindmarsh Island. Permissions sought and gained were conditional on a bridge being built from the mainland and this was objected to by local Aboriginal people on the ground of the cultural significance of the site. A reader unfamiliar with Australian society and politics might apprehend a ghost of that confrontation in a comment by Gummow and Hayne.

> There is an issue on the pleadings (so the matter cannot be assumed by the Full Court) whether the areas to which the Bridge Act applies are of a high spiritual importance to the Ngarrindjeri people and whether the building of a bridge would desecrate their traditions, beliefs and cultures (561).

The spiritual significance alluded to was for Ngarrindjeri women.[11] It had called out scandal at the very idea of law and government being asked to accommodate gender specific knowledge and in circumstances of internal conflict within the Aboriginal community, a Royal Commission of the State of South Australia aimed at establishing the 'truth' of the objecting women's spiritual beliefs had run its sorry if farcical course.[12] The women concerned took no part in it and I do wonder whether anyone seriously thought they would. But perhaps I underestimate the way in which belief in the cultural superiority of Europe corrodes the very reason of that culture. In any case, the record of protection of Aboriginal heritage afforded by Federal and State legislation claiming that purpose does not speak well for the capacity of the Australian legal system to realise its stated purposes.[13] And here again, one must wonder whether it was ever seriously intended that it do so. The Act confers procedural but no proprietary rights: rights of a kind which do not activate the common law's concern for its traditionally favoured subjects, the men and women of property. This aspect of the case is just below the surface of the judgements of Gummow and Hayne. Brennan and McHugh seem to have difficulty in recognising deprivation of rights other than proprietary rights as amounting to discrimination at all.

This absence of the meaning of the litigation for the plaintiffs goes to the formalisation of modern law referred to in the Introduction. The

[11] For a history of the region see [Watson 2002] and on this issue, [Watson 1997] esp at 49f; [Watson 1998] at 30f; for an anthropological study see [Bell 1998]. A chronology of the dispute is given by Bell at 641–646.

[12] See [Harris 1996].

[13] [Goldflam 1997]; more generally, [Finlayson and Jackson-Nakano 1996].

effect of the case by case aspect of common law praxis concerns issues which, according to the texts of the judgements were *not* before the Court. *Kartinyeri* was decided as legislation to amend the Native Title Act 1993 was in process of passage through the Commonwealth parliament. It became law later in the year as the Native Title Amendment Act 1998. 'Native title', a form of property in land, peculiar to Aboriginal people — not *of* the common law, since its source is said to be in the traditional laws and customs of the claimant Aboriginal group, but recognised *by* the common law — had become part of the common law of Australia in 1992 via the decision of the High Court in *Mabo and Others v The State of Queensland* (No.2).[14] Thereafter a regulatory regime was established by legislation, the Native Title Act 1993 (CW).

This Act had survived a constitutional challenge from the right brought by the State of Western Australia. Remarkably for those who had objected to the process of negotiating the passage of the legislation[15] and many of its provisions, the High Court held that the Act fell within the ambit of the race power as a "special law" that was beneficial to Aboriginal people.[16] That left open the issue of whether legislation deemed detrimental to Aboriginal people was authorised by the Constitution: the issue which was or was not before the Court in *Kartinyeri.* If it was and if the second argument on the race power in *Kartinyeri* were to succeed, there could not be much doubt that proposed amendments to the Native Title Act would be challenged. It had been openly said that the amendments would deliver 'buckets of extinguishment' of native title rights particularly over pastoral leases.[17]

[14] Above n.7; subsequently referred to as '*Mabo*'. There is a large literature covering a range of responses: from enthusiastic endorsement, e.g. [Bartlett 1993]; to outrage, see [Attwood 1996a] for analysis of these responses in terms of affront to white Australian identity; to more or less deeply sceptical analyses, e.g. [Mansell 1992]; [Kerruish and Purdy 1998]. [Motha and Perrin 2002] contains critical essays on the land/sovereignty nexus in the case. [Strelein 2006] covers native title cases since *Mabo*.. A concise and pertinent summary of *Mabo* is given in [Motha 2007] at 72f.

[15] See e.g. [Watson 1998] esp. at 39f.

[16] *Western Australia* v *Commonwealth ('The Native Title Act Case')* (1995) CLR 373. A summary of an admittedly complex case is found at `http://www.ags.gov.au/publications/agspubs/legalpubs/legalbriefings/br20.htm`.

[17] In December, 1996 in *Wik Peoples v Queensland* (1996) 120 ALR 129, the High Court by a narrow majority cautiously extended the common law principles of native title to envisage shared rights over land subject to pastoral leases. The decision

It cannot be said with textual warrant that the politics of native title determined or even shaped the decision in *Kartinyeri*. It is extremely hard to believe that it did not. The effect of the decision on the political context however is not in dispute. The decision affected the timing and micro-politics of getting the amending Bill through the Parliament.[18] The more reason then for taking a close look at the reasons for judgement in the case.

3. Analysis of the Judgements in *Kartinyeri*

For Brennan and McHugh interpretation of the race power was not before the Court. As I have said, the principle working their judgements and the dispositive part of Gaudron's judgement is that a plenary power to enact a law carries with it the power to amend or repeal that law. A general rule is cited from a textbook on British constitutional law.

> One thing no parliament can do: the omnipotence of parliament is available to change, but cannot stereotype rule or practice. Its power is a present power, and cannot be projected into the future so as to bind the same parliament on a future day, or a future parliament.[19]

Paradoxes of omnipotence with their theological accompaniment are famous,[20] but I don't want to rush into that just yet. What makes sense of this 'present power' is that it can be exercised with reference to and on the Acts of its past exercise but not with reference to itself. It thus remains 'available to change'. Given a written Constitution in Australia the question on which the judges divide, three to three, is how this present power stands in relation to it. Even so, a decision is reached, five to one: the Bridge Act is valid. Closure of the system so that it finds this answer from within itself (reflexivity or legal self-reference) must occur at some point. The disagreement is on where, in the chain of authorisation thought

triggered unprecedented attacks on the High Court in the media, the profession, the universities, the mining, pastoral and tourist industries and the governing coalition parties. See generally [Brennan 1998]; [Hiley 1997].

[18] [Brennan 1998] at 76f.

[19] Anson, *Law and Custom of the Constitution* vol.1 at 7 cited by Brennan and McHugh at 550.

[20] See e.g. [Frankfurt 1964].

to confer validity on laws, that point is. But the means of closure is the same technical operation in two of the judgements, that of Brennan and McHugh and of Gummow and Hayne: an assertion that an amending Act has one and one only consequence, which is to say that it has *no other effect* but to amend another law. This assertion takes the amendment purpose as determinative. The consequences of the amendment don't count.

Brennan and McHugh's argument is nicely represented by Kirsty Margarey as "a classic syllogism'.

a: the Commonwealth Parliament had the power to enact the Heritage Protection Act; and

b: the Bridge Act was an 'indirect express amendment' of the Heritage Protection Act effecting a partial repeal of the Heritage Protection Act; so

c: the Commonwealth must have power to pass the Bridge Act.[21]

For Gummow and Hayne the assumption in b) that the Bridge Act effects a partial repeal of the Heritage Protection Act is question begging. Considerations of amendment and repeal in their view, bear upon but cannot be determinative of the question because, the Bridge Act, if invalid, effects nothing at all.[22] They in no way set aside, the 'rule' that what parliament may enact it may amend or repeal. It is accepted and set out as a basic proposition of law relevant to the case. In agreement with Brennan and McHugh, they recognise that, contrary to the plaintiff's submissions, the effect of invalidating the Bridge Act would be a form of entrenchment. It would

> deny to the parliament the competence to limit the scope of a special law by a subsequent legislative determination that something less than the original measure was necessary (568).

But unlike Brennan and McHugh they admit the possibility that in the particular circumstances of the Australian Constitution, including the 1967 Constitution Amendment Act, it could turn out that the Bridge Act fell outside the race power. Indeed it was this possibility that called for interpretation of that power.

[21] Parliament of Australia, Research Note 41 at 1: `http://www.aph.gov.au/library/pubs/rn/1997-98/98rn41.htm` (accessed 24/04/2008).

[22] "If the Bridge Act be invalid, the operation of the Heritage Protection Act has continued unaffected by it" (561); and "If it be invalid, then there is no scope for the process of conflation [of Act and amending Act]" (565).

If it is not the nature of plenary parliamentary power from which the disagreement stems, nor is it the legal test for determining the constitutional validity of an Act. An authoritative formulation of the test is agreed and cited in both judgements from the same source. It is to determine the constitutional character of a disputed Act in terms of its "operation and effect, if valid", and this requires identification of "the nature of the "rights, duties, powers and privileges" which the statute under challenge "changes, regulates or abolishes" ".[23] And while, to this formulation Brennan and McHugh add, that in order to ascertain these rights, duties etc., an Act's "application to the circumstances in which it operates must be examined" (547), the addendum is not controversial. Even so, whereas Gummow and Hayne consider the effect of the Bridge Act on the rights, duties, etc. of the parties to the dispute, and conclude that it discriminates adversely against the plaintiffs, Brennan and McHugh preclude such considerations.

> Once it is accepted that s 51(xxvi) is the power that supports Pt II of the Heritage Protection Act, an examination of the nature of the power conferred by s 51(xxvi) for the purpose of determining the validity of the Bridge Act is, in our respectful opinion, not only unnecessary but misleading. It is misleading because such an examination must proceed on either of two false assumptions: first, that a power to make a law under s 51 does not extend to the repeal of the law and, secondly, that a law which does no more than repeal a law may not possess the same character as the law repealed. It is not possible, in our opinion, to state the nature of the power conferred by s 51(xxvi) with judicial authority in a case where such a statement can be made only on an assumption that is false (551).

Disagreement here is phrased in terms of the requirements of judicial duty, related back to grounds of decision. A political disagreement within the court on legitimation strategies sits alongside those grounds, implicating the articulation of the political to the juridical. Where, as in these two judgements, the approach plays up formal aspects of legal discourse, the disagreement appears as a classification issue.

Thus, Brennan and McHugh, via a classification of the Bridge Act as an 'indirect express amendment' of the Heritage Protection Act place it

[23] Gummow and Hayne at 562; Brennan and McHugh at 546-7 citing Kitto J in *Fairfax* v *FCT* (1965) 114 CLR 1 at 7.

within a class of Acts which refer to and effect only other Acts. What the Bridge Act *does*, (another formulation of its 'operation and effect') *and all that it does*, is limit the scope of the Heritage Protection Act. It effects a partial repeal of the Heritage Protection Act and that is its "only effect" (548; 550). Gummow and Hayne, in contrast, remove it from that class.

> The Bridge Act is not within that class of statutes which makes textual changes to the principal statute, so that it is "exhausted" upon its commencement and the incorporation of textual changes (565).

For them, all that we have, at this point, is a law which certainly refers to the Heritage Protection Act "but which does not identify the text it amends" (656). In the result there is an *interpretive* need to conflate the two texts in order to arrive at their combined meaning, but the Bridge Act has the character of a law effecting rights, duties etc. of persons and its constitutional character must be determined by examination of these effects.

The conundrum of conceptualisation can be put as follows. Does the plenary character of the legislative power conferred by the Constitution on the Commonwealth parliament condition the various heads of power or are these heads of power a condition of plenary legislative power vesting in the Commonwealth? Alternatively: are the various heads of power conditioned by or conditions of plenary legislative power conferred by the Constitution on the Commonwealth parliament? If the former, then given the power to amend or repeal inhering in plenary legislative power, each head of power is in effect a power to legislate in respect to (subject matter) X *and* to amend or repeal a law made with respect to X even if the latter is not itself a law with respect to X. This is Brennan and McHugh's view to which Gaudron would add the further clause: provided that as amended the principal Act remains a law with respect to X. For Gummow and Hayne the latter alternative holds so that the conjoined power of enactment and repeal is not yet operative.

They therefore consider that it is necessary to interpret the race power. This they do in a way that allows adversely discriminatory laws albeit within limits. Extreme examples, imaginable from "the lessons of history (including that of this country)", cannot be permitted to control the meaning to be given to federal legislative power in accordance with received doctrine. However, the need for clear and unambiguous language to effect an abrogation of fundamental common law rights, the power of judicial review vested in the court under the doctrine of *Marbury v Madison* and the assumption that the rule of law forms part of the Constitution as stated by Dixon J. in *Australian Communist Party v Commonwealth* (1951)[24] set limits which may, some day, have to be considered (568–9).

What remains now for Gummow and Hayne is the arguments based on international law and it is here, at a point where possibilities of an interpretive opening to cosmopolitan law flicker, that they use the technique of closure identified and extinguish them. The arguments as mentioned went to construction of the Constitution and the Constitution Amendment Act of 1967. The argument put by the Human Rights and Equal Opportunity Commission (for obligations of a legal character on members of the United Nations to protect human rights and a Constitutional construction appropriate to such obligations) fell to the ground that the Constitution is the supreme law of an "autonomous government" conferring on it plenary legislative power. Dixon is cited in support:

> Within the matters placed under its authority, the power of the parliament was intended to be supreme and to construe it [a section of the Constitution] down by reference to the presumption is to apply to the establishment of the legislative power a rule for the construction of legislation passed in its exercise. It is nothing to the point that the Constitution derives its force from an Imperial enactment. It is none the less a constitution (572).[25]

The problem now is that the plaintiffs' argument on the 1967 Constitution Amendment Act might seem to have support from the passage cited. Earlier in their reasons, Gummow and Hayne admit the existence of conflicting views as regards the effect of the 1967 amendment on the interpretation of the race power. And they have accepted

> that a statute of the Commonwealth or of a State is to be interpreted and applied, as far as its language permits, so that it is in conformity and not in conflict with the established rules of international law (571).

[24] Legislation proscribing the Australian Communist Party declared invalid because unauthorised by the defence power under which it was made.

[25] Dixon in *Polites v The Commonwealth* (1945) 70 CLR 60 at 78.

Do not then the admittedly conflicting views on the effect of the 1967 Act on s 51(xxvi) indicate that its language does indeed permit the rule of construction to be applied? No. That equivocation concerns s 51(xxvi) as amended and not the Act which amended it.

> A proposed law for the alteration of the Constitution passed in accordance with the special manner and form provisions of s 128 differs in character and quality from laws passed under the heads of power in ss 51 and 52. Upon the satisfaction of the requirements of s 128 . . . , the proposed law is spent and by force of s 128 the Constitution itself is altered. "*Its only operative effect* [was] to alter the Constitution, that and no more" (my italics: 572).

Gaudron's *obiter* opinion on the race power fastens on the political judgement that Parliament must make in 'deeming it necessary to make special laws' and affirms the court's supervisory jurisdiction over the exercise of that power in order to prevent its manifest abuse. The principle for the exercise of this jurisdiction — there must, in the circumstances of the time, be some relevant difference between the people subject to the legislation and other races to which the special law is reasonably adapted and appropriate — is however not operative in this case. In substantial agreement with Brennan and McHugh, she finds that the Bridge Act merely limits the field of operation of a beneficial law and remains, as an exercise of plenary power, within the constitution.

Kirby joins Gummow and Hayne in saying that the case *is* about the race power. What for Brennan and McHugh is an established rule, for Gaudron is in the nature of Commonwealth plenary legislative power and for Gummow and Hayne is a 'basic proposition' — that what parliament enacts it may repeal — becomes in Kirby's judgement first a 'maxim' and then an 'aphorism'.

> The aphorism that "what parliament may enact it may repeal" must give way to the principle that every law made by the parliament under the Constitution must be clothed in the rainments of constitutional validity (602).

Kirby J's reasoning takes the form of confessing the force of the arguments for holding the Bridge Act valid ("for a time they held me") and avoiding them by a string of reasons that moved him to conclude to invalidity of the Bridge Act (summarised at 593). I don't pursue his dissenting judgement much further here. Dworkinian in its jurisprudence and style it raises questions of the perpetuation of law's liberal promise and the function of dissenting judgements in the common law tradition. I do however remark one particular passage in relation to such questions.

One of Kirby's string of avoiding reasons is the unworkability of a 'manifest abuse' test, proposed by counsel for the Commonwealth. Using Nazi Germany and apartheid South Africa as illustrative cases of 'wicked regimes', he argues the inherent instability of the test: that beginnings of gradually escalating discrimination may fail such a test while termini may exceed a complicit judiciary's capacity to influence matters. A beneficial construction of the race power, he reasons, is mandated by these lessons of history, as also by "the experience of *other places* where adverse racial discrimination has been achieved with the help of the law" (598, my italics). It could be that the italicised words do not have an exclusionary intent. In the following paragraph, he writes:

> The laws of Germany and South Africa to which I have referred provide *part of the context* in which para (xxvi) is now understood by Australians and should be construed by this court. I do not accept that in late twentieth century Australia that paragraph supports detrimental and adversely discriminatory laws when the provision is read against the history of racism during this century and the 1967 referendum in Australia intended to address that history. When they voted in that referendum, the electors of this country were generally aware of that history. They knew the defects of past Australian laws and policies. And they would have known that the offensive legal regimes in Germany and South Africa under apartheid were not the laws of uncivilised countries (ibid, my italics).

The suspicion remains that for Kirby racial discrimination was not achieved 'with the help of the law'. There were defective laws then but not a record of *the* law's complicity in structuring Australian race relations. One point here is the distinction between 'civilised' and 'uncivilised' countries with all the unhappy and contested history of the distinction. But secondly, to stay just short of raising the history of modern European and in particular British colonialism as counterweight, there is the role assigned in this reasoning to the 1967 Constitution Amendment. It might be said that it *could have been* the kind of event in the nation's constitutional history that Kirby is saying it was. Alternatively it might be asked *could it ever have been* such an event? "Revolutions" Marx commented, "are not made with laws."[26] That might be thought to be too short, but if one does stop short of the history of British colonialism and the nation states that emerged from it, then these questions must stand unanswered. Thirdly, this text has its jurisprudential context. Nazi Germany and apartheid South Africa as 'wicked regimes' figure prominently in a jurisprudential debate of the last century on the relation between law and morality which pitted positivist against natural law theorists in argument about the concept of law.[27] Recycled here, it continues that debate.

What exactly its stakes are from the perspective of its participants I do not venture to say. From mine it looks like a thoroughly collegial discussion which sets up parameters of justification and legitimation. Its collegiality — not always amicable — is its institutional aspect. It delimits the range of relevant considerations, arguments, feints and guises that may be brought to the debate. 'Wicked regimes' it would seem are a sustaining feature of it. And it is nothing if not 'reflexive', meaning that if it encounters serious challenge, it revises its determination of relevance to include the challenge.

4. Justification and Legitimation

> Here [language] has for its content the form itself, the form which language itself is and is authoritative as *language*. It is the power of speech, as that which performs what has to be performed.[28]

Consider a fancy. What would have resulted from Gummow's and Hayne's approach in *Kartinyeri* had they interpreted the race power to permit only beneficial legislation? The plaintiffs would have succeeded (by virtue of the interpretation) and, further, given that the written text of the Constitution conditions or controls Commonwealth legislative power, 'special laws' would gain a form of entrenchment. Repeal or amendment of laws passed under the race power 'for the benefit' of Aboriginal peoples would be liable to challenge. Aboriginal people would thus gain a participatory

[26] [Marx 1976] at 915.
[27] See e.g. [Hart 1961] at 195f; [Fuller 1969] at 159f.
[28] [Hegel 1977] at 308; [Hegel 1807] at 390.

power, exercisable through a politics of intervention, in the determination of which laws *are* for their benefit and for the maintenance of beneficial laws. That would have given them, minimally, a special place in the Constitution: a place not of being “done to” in Kevin Gilbert’s memorable phrase,[29] but for exercising a supervisory power over legislation passed under the race power ‘for their benefit’.

I do not want to take this fancy as revealing yet another potentiality within law to respond somewhat more graciously to its subjects. I would wish too that its difference from imagining Kirby’s dissent as a majority decision be observed. The fancy is an artifice. It imagines an outcome which *none* of the judges were willing or able to reach, by the unlikely combination of Gummow’s and Hayne’s deferred application of the closure operation identified together with the beneficial interpretation preferred by Kirby and Gaudron. Call it a ‘thought experiment’ if you will. It is a device for exploring how and why this outcome although imaginable, is *in some way* specific to the Australian Constitution not constitutionally imaginable.

I think it uncontentious that the practice of giving reasons for judgement is both justificatory and legitimative; that although presented as if working their way to a conclusion (the decision: the Bridge Act is valid/invalid), the decision has been reached beforehand. In this section I go back over the judgements as exemplars of judicial praxis and its techniques — exercises of *tēchné*, that skilful doing that can deceive the eye and is part of the practice of a craft[30] — examining first the intentions of the decisions justified[31] and then the legitimation strategies deployed, all the while aiming at specifying that ‘some way’ in which the fancied outcome is constitutionally impossible.

It should go without saying that the outcome of my fancy was never an open possibility for Gummow and Hayne: never lay within their intentions. They cite authority (in the sense of decided cases) of their own court, to make the “general conception of English law that what Parliament may enact it may repeal” a ‘basic proposition’ of Australian

[29] [Gilbert 1994] at 13.

[30] [Kerruish 2002]; and for a brief account of episteme and techne in Greek antiquity which informs my use, see [Russo 2004] at 185f.

[31] Meaning here to distinguish intentions from motivations: so ‘intentions’ as evidenced by the decision and its justification.

constitutional law (562). Their deferred use of the closure operation defeats and was always intended to defeat, an argument from international law which on their own admission is part of Australian law. Brennan and McHugh use the same technique[32] at an earlier point to further their intention of deciding against the plaintiffs without interpreting the race power. And likewise, despite differences in jurisprudence and decision, for Kirby: the form of entrenchment in the imagined outcome never lay within his intentions. It is, on the contrary, explicitly denied.

If disagreement here appears quite variously — as a disagreement on what the case is about; as a classification issue; as a disagreement concerning judicial duty; as a disagreement on consequences — that is unsurprising. It concerns the exceedingly subtle question of how a unified and anterior common law informs a written Constitution (Australian) deriving its force from a statute of a parliament (United Kingdom) the unlimited sovereignty of which is a creature of English common law.[33] In other terms, the disagreement is located in the misty, not to say mystical or magical,[34] regions of the authority and force of the Constitution.

As regards its force Gummow and Hayne take their stand from Dixon.

> It is nothing to the point that the Constitution derives its force from an Imperial enactment. It is none the less a constitution.[35]

What the Imperial authorisation is nothing to the point of is the distinction Dixon has just made between the establishment of legislative power and the exercise of that power. It is a slight variation on the distinction that diverts common law notions of parliamentary omnipotence from the

[32] A distinction creates or supposes two classes of rules (principal and amending) and selects the latter as a domain for application of a strict rule: a Constitution amending act/amending act has one and only one effect.

[33] I take this formulation from a reading of [Dixon 1965]; see also [Veitch et al. 2007] at 10f. If the issue is so subtle as to be called undecidable so be it, but I would not wish to assert an analogy here with undecidability in formal mathematical logic: a parallel rather and a gap, conventional and continuously reiterated, between mathematics and both philosophy and the other sciences ([Harris 2008]). I am aiming at a notion of the wrong of law that draws on undecidability in its formal mathematical context and my point here would be to portray any analogy more specifically.

[34] The references are on the one hand to [Derrida 1989] and on the other to [Ross 1969].

[35] Above n.25.

old paradox of divine omnipotence, maintaining the former as a 'present power'.[36] Authority comes from the common law. Brennan and McHugh occupy the very same ground, but position themselves otherwise, so that the force of the Constitution needs no separate mention. They call up the veritable divinitude of a power "so transcendent and absolute, as it cannot be confined whether for causes or persons within any bounds" (548).[37] Invoking this "sovereign and uncontrollable authority. . . "(549)[38] and bound to the observance of such a mighty power, judicial power is represented as being exercised in straitened circumstances. Their intention can be redescribed as being to dispose of the plaintiff's complaint as economically as is possible. Substantive considerations are what is to be economised on. The guise of a logic of correct legal reasoning comes in aid. The decision is, in effect (and in likely motivation[39]) that the complaint has no substance. As logically necessitated the reasons for judgement can be given the form of a classical syllogism.

Recalling the rhetorical ease of Kirby's invocation of principle to free Australian law from an inappropriate maxim and noting that the historical appropriateness of its rules is a criterion of the authority of the common law in classical common law theory,[40] brings his stance on this subtle question into the picture. He might take the stance that legal reasons "are best understood as asserting moral claims",[41] or, more likely as it seems to me, that because his decision does sustain law's liberal legal promise it is the right answer in this case.[42] I will not further explore the recessive spaces of this question. I have said enough I hope to show how accommodating they are.

[36] Above at 9.

[37] Citing Blackstone as adopting the views of Coke.

[38] Citing Blackstone's further commentary.

[39] Whereas Gummow and Hayne admit the removal of procedural rights from the plaintiffs, Brennan and McHugh observe that no proprietary rights are at stake (545).

[40] [Postema 1986] at 4–14.

[41] See [Coleman 2007] at 14, n.4.

[42] Closer then to Dworkin's idea of law as integrity than to so called 'inclusive legal positivism'. Morality here becomes political morality, law is an 'interpretive concept' and closure, interpretive rather than classificatory, comes to rest on the assertion that "[a]ny political theory is entitled—indeed obliged—to claim truth for itself, and so to exempt itself from any skepticism it endorses" ([Dworkin 1985a] at 350; [Dworkin 1986] at 108f.

They are accommodating enough to enable *all* the judges, despite their disagreements, to stay within formal limits (or bounds) of judicial competence. As *judicial* competence it is distinguished from legislative competence (separation of powers), while as competence or power it is constituted by its limits. By convention and as conventionally described these limits permit the exercise of a supervisory jurisdiction in accordance with the *Marbury* v *Madison* doctrine of judicial review.[43] Practically it is mind-numbingly predictable that the judges will present their reasons as being within the limits of their competence. So far as individual judges are concerned they have made their way to the top and are masters of their craft. So far as law and the constitution are concerned the limits are just those of the authority and force of law discussed in the previous two paragraphs. But there are two rather less obvious points to be made here.

First, Gummow's and Hayne's reasoning *like* Kirby's and *unlike* Brennan's and McHugh's (and Gaudron's in effect) *expands* judicial competence by holding that Commonwealth plenary power is subject to the written Constitution. The powers are enumerated, the realm (of Commonwealth legislative power) is finite and it falls to the judicial power to keep the legislature within the limits set by interpreting the text of the Constitution. Thus for both it is right and proper to interpret the race power. True enough, Gummow and Hayne would likely say that this formulation leaves out what is critical in their reasoning: this power is finite but it is supreme. Interpretation should not transgress this 'basic proposition' since that would be an 'error of law'. Let me leave that run. The second point is that 'staying within' formal limits of judicial competence is exercising a full gamut of powers constituted as 'judicial competence' by the constitution (not just the written Constitution) as 'foundation' of a polity. This is a loaded point at which the device of my fancy comes in aid. The fancied outcome has the *effect* of giving over to Aboriginal people a supervisory power over legislation passed 'for their benefit'. None of the judges do this. Were it appropriate to speak of the will of the Court as a whole — a seemingly fictitious notion — it could be said to be set against the imagined outcome. The judges however would say, with every

[43] It was assumed from the beginning (i.e. the Constitutional Convention debates) that the High Court, like the US Supreme Court, would as interpreters of the Constitution, have the power to invalidate Commonwealth and State legislative and executive action ([Hanks 1991] at 22).

right, that they *can* not do it; that they *must* stay within the limits of judicial competence. Which, I suggest is also to say that they must decide the case in such a way that no modicum of power over their own lives is returned to Aboriginal people.

We should look then to the sense in which they *can* not do it. The judges might say, in a Kantian idiom, that they were 'entitled and indeed obliged' to decide as they did.[44] It is the latter aspect, the appeal to necessity, *qua* obligation, that the 'can not' speaks out. I do not want to leave this jurisprudential sense out of account. It is part of what is going on in this case; in any case in so far as the necessity averred is a way of saying what counts as legal reasoning. The distance taken by looking at the judgements as exemplars of a practice cannot avoid this if, as seems to me to be hermeneutically required, the understandings of participants in the practice is to be taken into account. Yet if the outcome of the fancy is excluded for all the judges, they have differing ideas as to what, concretely, is required of them. To get at that, I suggest, account should be taken of the sense in which, in addition to being justificatory (of the decision), legal reasoning is legitimative of the law on which the decision is grounded.

The legitimation function in the practice articulates law to legitimate political power. In one way of looking at it it works by selecting and plugging in various 'arguments' — narratives — which will return a determinate 'positive' value (responsible government, representative democracy, equal enjoyment of rights e.g.). In another it gives occasion to pursue various legitimation strategies.[45] Either way, looked at from the perspective of legitimation, judicial disagreement goes to which argument or which strategy will best serve the needs of the moment as perceived by the judges.[46] In circumstances in which the authority and efficiency of the

[44] Kant applies this idiom (*berechtigt, ja verbunden ist*) to formal logic's having, as the condition of its success, to abstract from all objects of knowledge and their differences ([Kant 1929] at 18; [Kant 1781/87] at 15.

[45] Bert van Roermund develops a theory of law as "a kind of self-questioning conceptual discourse" from a logical and epistemological analysis of the intersection of conceptual and narrative discourse at various levels of law's social and institutional *Dasein* ([van Roermund 1997] at 16). While my aims and approach are different and probably incompatible with his, I am indebted to his work for aiding my understanding of conceptual and narrative components of legal discourse.

[46] I have used a formulation which leaves open what or whose needs are thus served. That will depend on the narrative chosen, the strategy pursued.

Court had been questioned Brennan and McHugh opt for legitimacy conferred 'time out of mind' on their authority or the authority of the court by the narrative of the common law itself.[47] Thus it is Coke and Blackstone who stalk their pages. And thus too they represent themselves as duty bound not to go beyond premises of common law origin. Gummow and Hayne, on the other hand, are pursuing that side of the colonial experience which emerges, gradually or tumultuously, as the colonist asserts autonomous national identity against the colonial/Imperial power. Perhaps they could be said to be making good for a declaration of independence that did not take place in revolutionary style.[48] Their legitimative strategy has the guise of neutral, distanced description of 'the law as it is'. In the narrative appropriate to their approach, Coke and Blackstone slip into the recesses of distant memory, their place taken by the acts and decisions of Australian parliaments and courts and the words of its celebrated jurists, in particular Sir Owen Dixon. A justice of the Court from 1929–1952 and its Chief Justice from 1952–1963, he defended a "strict and complete legalism" as the only "safe guide to judicial decisions in great conflicts".[49] Certainly they are not looking the gift horse of the older common law narrative in the mouth. But there is an ongoing task of its patriation.[50]

Abstractly considered, Gummow and Hayne could, like Kirby, have opted for a different, but still national post-colonial narrative with the 1967 Constitution Amendment Act and its accompanying referendum at

[47] [Kerruish 1998] at 72f, drawing on Postema's text on the classical common law tradition ([Postema 1986] c.1.

[48] Cf. [Motha 2002] interrogating judicial pronouncements of the non-justiciability of sovereignty in *Mabo* and locating ambivalence, after Derrida, in the undecidability of constative and performative aspects of declarations of independence.

[49] [Dixon 1952] at 247; and see [Hanks 1991] at 21–26 for brief discussion.

[50] 'Patriation' is more commonly applied to constitutions which, as enactments of an imperial legislature, are to be brought home to the newly autonomous state. I am using it here to refer to the conversion of English to British to Australian common law. In contrast to the United States of America (*Erie Railroad Co. v Tompkins* (1938) 304 US 64), the common law is not doctrinally scripted to form a separate system of jurisprudence in each of the Australian states. The prevailing view, stemming from Dixon, is "that the common law is one entity" ([Sykes and Pryles 1991] at 332; [Dixon 1957]. It is cogently questioned by [Purdy 2000–2001] at 70. It seems to me to be a second line of defence of that 'unity' of sovereignty that works against recognition of Aboriginal law.

its centre. But that is an option appropriate to legitimation in terms of substantive liberal principles (Kirby), not to re-establishing the authority of the High Court via notions of judicial objectivity and political neutrality afforded by the "technique of the common law" and "the use of the logical faculties".[51]

This is the occasion for returning to the question left run regarding the difference between Gummow and Hayne and Kirby. They are agreed that Commonwealth plenary legislative power is subject to the Constitution. But on Gummow's and Hayne's reasoning Kirby erred in accepting counsel for the plaintiffs' submission that their position did not entail judicial limitation of parliamentary competence. Kirby's counter argument, the necessity that 'laws be clothed in the rainments of constitutional validity' is as undoubted in a constitutional democracy as it is vacuous given argument about what these rainments are or should be. It may sound somewhat more 'literary', more 'extravagant' or 'metaphorical' than the necessity of determining meaning "in accordance with received doctrine"(569), which is Gummow and Hayne's ground for rejecting Kirby's view, but there is room for scepticism there too. Is there a received doctrine of constitutional interpretation? A received doctrine for deciphering the effect of Aboriginal people being, in the cited text of the race power, written under erasure? Or is 'received doctrine' a figure of speech signifying Gummow's and Hayne's view on methods of or approaches to constitutional interpretation given that their legitimation strategy is return to an earlier era of legalism? I do not wish to suggest with this questioning that received doctrine is no part of legal reasoning. I think it is. That gives all the more occasion for passing off disagreement as doctrinal error: justification and legitimation rub up against, intrigue with each other.

On the other hand there is a not quite symmetrical obverse of the claim that Kirby's judgement is wrong in law, namely that it is the best or the right answer in the case *because* it shows Australian law in its best possible light or *because* it affirms modern law's commitment to equal rights. The justificatory argument on the entrenchment issue is that the risk of irresponsible exercises of parliamentary power outweighs the maxim that what parliament enacts it may repeal (602). Justification and legitimation work together to suppose a morally (or politico-morally) ideal realm

[51] [Dixon 1955] at 165.

within which the legalistic argument that the 5:1 division shows Gummow and Hayne to have been right on the entrenchment issue and Kirby wrong is inverted. Yet Kirby, in agreement with the other judges, will not or cannot admit to returning any modicum of power over their own lives to Aboriginal people. Judicial competence is expanded. The written text of the Constitution is supreme. But the denial of entrenchment in Kirby's judgement is, performatively, a refusal to confer political power on Aboriginal people. The judge holding up the beacon of dissent takes the powers to and for himself as judge, as law-sayer, so that the justice of equal rights may be 'done to' Aboriginal people.

Where does this leave us? Immersed, I would say, in jurisprudential controversies. It looks as if, if one is to take account of what is going on in the case, it is not possible to get out; not possible to think the necessity that translates into the constitutional impossibility of my fancy in any terms other than these various views on and performances of judicial duty. It looks like that. To a degree it is like that. Within contemporary jurisprudence in its conventional shape it is probably right to say that what makes my fancy a fancy is the combination of techniques, approaches and styles which do not go together, that compete in their conceptions of law (and consequently of judicial duty) albeit from a shared concept of law.[52] We come back here to points touched on previously: most generally at the end of my Introduction as the effectiveness of judicial praxis in spinning the stuff and matter of social life into the gold of doctrine; again at the end of Section 3 as the collegiality of jurisprudential argument on law and morality; and, as a matter of method, in my comment in this section on the limitations of a practice perspective.[53] What I would now add is that it is more the legitimative aspect in the guise of judicial duty or obligation than the justificatory aspect of the practice which leads to this impasse, although given the ways in which justification and legitimation intrigue this point is difficult to recover. Still, I would say that while, in a doctrinal discourse, justification as principled is conceptual, learnable and deconstructible, legitimation 'performs what has to be performed'.

[52] [Dworkin 1986] at 70f.
[53] Above at pp. 4, 16, 21 resp.

5. Imperatives of Extinguishment

In this context that is to legitimise a take-over: to substitute the common sense of the common law for the cosmological sense of being in place and time that informs or is Aboriginal law. To my mind, what is happening here is that the *work* of extinguishing Aboriginal law is being promoted by belief in and arguments for the necessity of the official voice stemming from the official voice. To this, writing from within the culture which has promoted the take-over, my objection is that this form of legal self-reference trades the actuality and presence in modern law of the official voice for various stories, all re-played from classical English common law theory, of the legitimate authority of law. If we go back here to the centre point of Gummow and Hayne's legitimative argument,[54] and to the assertion that "the occasion has yet to arise for consideration of all that may follow" from Dixon's affirmation that the Constitution assumes the rule of law (569) then, bearing in mind the many and diverse occasions on which Aboriginal people have sought recompense for harms done to to them and failed, it becomes apparent the such harms do not count, present no such occasion to this law and its legitimising notion of fairness. A fair English skin would seem to be the effective criterion although one knows that that too is a joke.

Veitch's thesis of how law works to disappear responsibility for massive harms, and specifically, his analysis of Brennan's absolution of the common law's responsibility for the dispossession of Aboriginal people is pertinent here.[55] Without doubting that modern law can and does distribute responsibility for harms done and suffered, he directs his inquiry to "the ways in which legality *can* and *does* allow the production of suffering" and against "the 'common sense' assumption" that the infliction

[54] Above at p.12.

[55] [Veitch 2007a] at 106f; [Veitch 2007b]. Brennan wrote: "Aboriginal rights and interests were not stripped away by the operation of the common law on first settlement by British colonists, but by the exercise of sovereign authority over land exercised recurrently by governments.. . . Aboriginals were dispossessed of their land parcel by parcel, to make way for expanding colonial settlement. . . . Even if their be no such areas [where native title has survived extinguishment.VK] it is appropriate to identify the events which resulted in the dispossession of the indigenous inhabitants of Australia, in order to dispel the misconception that it is the common law rather than the actions of governments which made many of the indigenous people of this country trespassers on their own land" (*Mabo*, above n.7 at 50). See also [Purdy 2000–2001].

of massive harm is exceptional or excessive to the "rational and reasonable normality" of the rule of law.[56] I am placing that which, as regards Brennan's absolution of the common law, Veitch terms an "inability to have the question of responsibility raised at all"[57] and relates to the constitution and shaping of sovereignty through colonialism, into the trade mentioned. The difference, as far as I can see, is largely a matter of approach. Beginning as I do with the form of modern law, we are looking at the process and product of a discursive logic which endows law with the form of an ideally, abstractly equal exchange: a universal equivalent of persons.[58]

If the ingenious character of that trade or exchange is admitted; if the effectiveness of judicial praxis in spinning the stuff and matter of social life into the gold of doctrine is acknowledged then, curiously perhaps, the *content* of legal doctrines now standing in the place of the vanished materiality of social life is the counter to the jurisprudential representation of the constitutional impossibility of my fancy.

Concretely, my hypothesis is that the impossibility of this outcome is vested in the twinned doctrines of the extinguishment of Aboriginal sovereignty on colonisation and the non-justiciability of this act of state in the courts of that state. The extinguishment doctrine is expressed in the proposition that

> the contention that there is in Australia an aboriginal nation exercising sovereignty, even of a limited kind, is impossible in law to maintain.[59]

The non-justiciability doctrine, while older than *Mabo*, is repeated there.

> "The acquisition of territory by a sovereign state for the first time is an act of state which cannot be challenged, controlled or interfered with by the courts of that state."

[56] [Veitch 2007a] at 10 and 19.

[57] Ibid at 107.

[58] Pashukanis' commodity form theory of law ([Pashukanis 1978] is not quite the theory endorsed here, but I think his perception of the significance of the form of law and of the close analogy of modern law with the 'logic of capital' as portrayed by Marx in his chapter on value is an insight. China Miéville's study of international law ([Miéville 2005]) has reminded me of the necessity to hold on to that insight if not to hold it bound to the premises of Marxist thought.

[59] *Coe* v *The Commonwealth (The Wiradjuri Claim)* (1993) 118 ALR 193 per Mason CJ citing Gibbs J in *Coe* v *The Commonwealth* (1979) 24 ALR 118.

> This principle, stated by Gibbs J in the *Seas and Submerged Lands* case, precludes any contest between the executive and the judicial branches of government as to whether a territory is or is not within the Crown's Dominions.[60]

In respect to the extinguishment doctrine, Australian history and jurisprudence is different from that of Canada, New Zealand and the United States of America. One might place that difference — an absolute non-recognition of indigenous inhabitants of a territory *as peoples* — into the context of a "national legacy of unutterable shame"[61] were it not for the brutal fact that it is not only uttered, it is *doctrine*. It is written and re-written into Australian law as the High Court reiterates, again and again, both the extinguishment and its jurisdictional inability to call into question the act of state from which its own authority derives. From *Mabo* on native title cases have been the occasion for this reiteration.[62] Re-visiting and revising the consequences of the take-over, Australian law recognises native title rights on that basis. They too are vulnerable to extinguishment by acts of government (the political sovereign). Protected by anti-discrimination laws, they become commodifiable: liable to compulsory acquisition on payment of 'just compensation'.[63]

The question that I am asking and have been asking throughout this paper is where is the imperative mood or modality of the legal assumption of the extinguishment of Aboriginal sovereignty and law hiding? In the course of hearing argument in *Griffiths*, the case just noted, Gummow in an exchange with counsel drily acknowledges the 'paradox' of property rights under native title and under common law being both juridically different and, by virtue of the human rights considerations of the *Race*

[60] *Mabo* above n.7 at 20 per Brennan. For a collection of essays interrogating this assumption of Australian law and its place in *Mabo* see [Motha and Perrin 2002].

[61] *Mabo* above n.7 at 79 per Gaudron and Deane; and see for commentary [Purdy 2000–2001].

[62] Most recently, to my knowledge, in *Members of the Yorta Yorta Aboriginal Community v Victoria* [2002] HCA 58 per Gleeson, Gummow and Hayne at [39]f, where the issue is euphemistically cast in terms of 'An intersection of two normative systems'.

[63] In *Griffiths v Minister for Lands, Planning and Environment* [2008] HCA 20, the High Court of Australia decided that, conditional on payment of 'just compensation', Crown land subject to native title could be compulsorily acquired by the Government of the Northern Territory for the purpose of selling or leasing the land to a private corporation.

Discrimination Act, equivalent.[64] The exchange is illustrative. The answer to my question is too well known. It hides in 'paradox': of justice, of authority, of sovereignty, of equal and unequal rights. And as the paradoxicality of it all settles into common sense (and there are many other terms that do the same work: 'irony', 'complexity' e.g.) yet another mechanism of legitimation is activated.

That raises further issues, but combining analysis and hypothesis, I am suggesting that the legal assumption of the extinguishment of Aboriginal sovereignty and law directs judges away from the outcome fancied. Hiding within the very notion of what counts as 'legal' reasoning, the imperative modality can be located in an 'internal point of view' or an 'interpretive attitude' or a 'realistic description of what judges do' and moved into epistemic or interpretive or pragmatic theories of the nature of law with this, that or the other degree of scepticism toward the authority of law: a nice issue for collegial discussion. Aiming at a concept of the wrong of law these designations appear as covers under which an assumption — an exceptional and for Aboriginal people still living in their law, false assumption — is constantly working to realise itself.

6. Concluding comments: on 'foundations'

My hypothesis then is that the 'some way specific to the Australian constitution' in which the outcome of my fancy is imaginable but not constitutionally imaginable is its inconsistency with the doctrines mentioned.

The constitution as legal foundation is written over thought's logical foundation, excluding the surprises that happen when thought in its being as being at odds with itself, trips over its own feet: that is, when it falls into logical paradox or antinomy. So far then from the idea of thought's logical foundation having an instantiation in constitutions as legal foundations, dialectical and speculative logical foundations are to my mind incompatible with them.[65] This thought runs toward distinguishing

[64] *Griffiths v Minister for Lands, Planning and Environment* [2007] 207 HCATrans 685.

[65] Since I take the idea of thought's logical foundation from Hegel it seems proper to note that as I read him, he perceives this incompatibility. In his *Elements of the Philosophy of Right* he resolves it by placing world history as judge of the nation state and, in his *Encyclopedia* by placing art, religion and philosophy, as manifestations of

different notions of 'foundation' in logically and epistemologically foundationalist theories. In particular, it sees a difference between a notion of (formal) 'logical foundation' as substrate that guarantees the certainty of established knowledge (say 'Kantian') and a notion of 'foundation' that inquires after dependencies (say 'Hegelian'), risking its own assumptions in that inquiry.

And the constitution as legal foundation is written over the historical, material circumstances and conditions of its own coming into being. It is, as a foundation, the foundation of a realm of thought, of ideas: ideas that are coercively enforced and insinuated certainly, but with a sufficient degree of autonomy to impose fictions — such as an Act having one and only one effect — in support of the order it maintains *without* dissolving into incoherence; *without* touching the official voice with doubt; *without* unseating it, so to speak. This thought of a double inscription, and again this is perhaps curious, runs toward enabling thought about law's foundations to affirm both somewhat Hegelian and somewhat Marxist — classically idealist and materialist notions of 'foundation'.

The force of law as the force of form allows force in the sense of physical and psychic violence to be exercised under a guise. In Bourdieu's terms, that guise disguises its "true nature" as force, gaining "recognition, approval and acceptance" by presenting itself "under the appearance of universality — that of reason or morality".[66] I agree regarding the force of form but differ from Bourdieu in so far as the rationalist passion of my foundationalist approach, differently to his (Pascalian) anti-foundationalist commitments,[67] sees reason as hollowed out, almost ruined — lamed in the face of fetish phenomena which it cannot handle and consoled by worshipping the shape they take as moral or legal values or validity — but still a force that can be set against physical and psychic violence.[68] Reason is lamed in and by its very *own* failure to break with 'the old wardrobe' of logical and epistemological ideas which cannot handle fetish phenomena. I do not follow Marx in his naming and analogy of these phenomena. I

absolute spirit above the constitution of the nation state, a manifestation of objective spirit. The labour of the concept is in time and history is not at an end.

[66] [Bourdieu 1987] at 85.

[67] Elaborated in [Bourdieu 2000].

[68] Under quite some conditions of which, as in [Kochi 2007], attention in ethical theory, to the relationship between thinking and action, negativity, positing and violence of creative-destructive and thoroughly material subjects, is one.

have argued that elsewhere.[69] But I think that his encounter with them in the context of political economy is an apprehension of thought's excess of the material, habitual, time-bound world of everyday life which presents theory with a task of logical investigation. It faces a barrier in a continuing hegemony of Aristotelian and Kantian ideas of formal logic as contentless, although it has been remarkably filled, from the turn of the last century, with surprisingly paradoxical results.[70] The 'irony', 'the paradoxicality of it all', phrases indeed empty of content, delivers reason from that investigative task to the service of common sense already formed by the law in force.

Reason is not ruined by thought's excess. It is freed from the enclosures it itself creates in its work of concept formation. It is ruined by belief in narratives of the cultural superiority of Europe. It is ruined by despising the material, habitual, time-bound, place-bound world of everyday life: despising its own conditions of being. It is ruined when justification ends its critical task in ideology or dogmatism: when for example, a rationalist passion claims for 'rationalism' (whatever that may be) an intellectual virtue over, say 'empiricism' or 'pragmatism'. It is not ruined by law. It has no one and nothing to 'blame' but itself for it is ruined in abdicating the task proper to its critical exercise, in self-celebration or diffidence or in crying impotence in the face of what it has been party to producing.

I have suggested that legitimation is more potent than justification in its arrest of thought about legal institutions, practices and their techniques, doctrines and categories and further that justification in moral or politico-moral terms only strengthens that potency. I have not written of how it is that a political will or conjunction of political wills directed to setting these Australian doctrines of sovereignty and jurisdiction aside does *not* gain the strength needed to remove the basis on which the legitimation strategies of *Kartinyeri* work. That is however a question which appears on the horizon of this study.

[69] [Kerruish 2007].
[70] [Kerruish and Petersen 2006]; [Petersen 2007].

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Having an Ethical Discussion About the Objects of Natural Science: The Is and Ought Distinction

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1. Telos and anti-telos

For Aristotle the question of the nature of nature was given an answer through his *teleology*. Every being had its own *telos*, its own "inner end" to realise. The realisation of this inner end was regarded as a *natural movement*. (*Physics* 194 a 30, 199 a 30; *Metaphysics* Bk. VII (Zeta) Ch. 17). Nature had a direction that humans could cross. Galileo Galilei challenged this teleology by showing that the stone was "searching" to the ground not because of an inner want/shortage (*steresis*), but because of an *outside* force (Barrows 1991, p. 18); a force that could be described through mathematics, and formulated as a law of nature. Beings of nature are on this view thought of as principally governed by laws of nature. Hence, natural beings could not be said to "have an inner end". As there only exist states that either govern or *are governed* — beings of nature have to be called hetero-nomous. Successively, from Galileo on, the being of beings was thought of as *passive* (Ellis 2002, e.g. p. 62; Szerszynski 2003, p. 152). Consequently, after the acceptance of Galileo's view, the idea that nature had a nature that could be crossed lost more and more ground. However, the mark of something having a nature was defined by

[1] Department of Philosophy, University of Tromsø. This paper has emerged from a presentation given at the annual workshop, 'Post-normal science and its ethical aspects: Doctoral projects and other projects in the making', at the ASFPG Centre for environmental and technology ethics. I would like to thank Matthias Kaiser for the initiation of this workshop and for his nice and smooth way of leading the workshops. I would also like to thank the participants at the workshop and last but not least, Valerie Kerruish and Uwe Petersen for very useful comments on the work in progress presented here, but also for making the workshops possible.

Aristotle's teleology. After Galileo it is only humans that could possibly live up to that standard.

This ontology not only comprises the "law governed sciences" but also biology in the following way. Biology tells us that there is no "right nature", "wrong nature" etc.[2] The biologist has no criterion to claim that there should not be "a desert in the Amazon Jungle" and she will never find such a criterion under biology's present ontological commitments. So even though biology is not law governed or law-like in any respect, it is still committed to what we could call a "passivist view on nature", borrowing a notion from Brian Ellis.

This is the ontological 'superstructure' or background that I regard as relevant to the issues that I will discuss and present in this paper.

2. "The value of passivism"

As Hume noted, we cannot *logically* infer from an *is* to an *ought* (1978, p. 469). The big mistake however, is to take this as an argument against any "permission" to follow an 'is' with an 'ought', or to say that we are committing a fallacious act as such when doing so. The view I want to convey here is that the desirability of following an 'is' with an 'ought' depends on ontology, and accordingly the intuition "that you cannot at all argue for what ought to be from what is", depends on our ontological assumptions. If nature were totally deterministic or totally contingent the intuition would follow, but given that we grant the possibility of other ontologies, there is nothing left in Hume's formulation other than triviality: very few non-formal facts, if any, outside logic itself and mathematics, can be logically inferred from something else.

As I have indicated, the assumption here is that there is a strong relation between ethics and ontology. To assume this is controversial. It is, however, not more controversial than what was fundamentally assumed by

[2] However in some sub-departments of biology there has indirectly been a debate going on about the nature of Nature. In ecology: natural balance in the ecosystems; in geophysics/ecology/meteorology: the Gaia theory (Lovelock 1979); in discussions about evolution: is it going towards diversification (Brown 2001)? in systematics: could there be something called an ideal individual of some species (Williams 1996)? Normally however, the reasoning goes as Reiss and Straughan represent it: species [Nature] change over time. It is not possible to distinguish between natural and artificial changes (1996, p. 61).

the seventeenth century philosophers and which we still take for granted, although we take it the other way around.

Kant assumed that since nature was governed by laws (*Naturnotwendigkeit*) humans must be governed by something else (1998, p. 52/4:446). The starting point for Kant's idea about autonomy (and heteronomy), which in turn came to be a part of the framing of ethics (what it is and can allow itself to be about) is grounded in ontology, that is, in *a specific account* of what nature is. To say that ontology should not impinge on our ethics is therefore strange, unless we accept that the specific law-governed-nature account is *the* account of nature. In other words: the norm that claims something to be a naturalistic fallacy, that is, the fallacy of thinking that *is* and *ought* are connected, is by its own measure a (meta-)naturalistic fallacy since it has itself emerged from a particular ontological assumption about the world. This fallacious view is of course also well established within environmental ethics, although this type of ethics typically tries to overcome traditional ethics (e.g. Curry 2006). In *Environmental Values* O'Neill *et al* make the following claim an important presupposition for their discussion. "No specific meta-ethical position is required by any specific environmental ethic" (2008, p. 119). This leads to assertions such as: "expressivism has no claims about what has ethical value" (ibid, p. 117).

My claim here is that we cannot grasp or get outside the "traditional human centred ethics" (ibid, p. 92) without touching upon the ontological roots of this ethic. To take the strong is/ought distinction for granted is to take the "human centred ethics" for granted. And moreover it is to take the passivist ontology to be *the* ontology. To believe that we can avoid metaphysics is a part of this same metaphysics. This background, hopefully justifies why we have to take a broad and "ambitious" view as a starting point for our elaborations.

I hope that this clarification will establish a ground for the postulation that I will maintain throughout this text. If our ontology will allow no causal powers to the beings of nature it will have some impact on our relation to those beings. No conduct, much less any moral conduct, can be attributed to something which is viewed as in principle passive. If this is the ontology of modern science it makes it *a priori* impossible to ascribe any value to nature as such. Putting nature together in new ways or changing conditions for the beings of nature will mean nothing ethically. If everything is disentangled and all identities are contingent, how can we possibly do any harm? Our ontology makes a difference to our ethics.

To science everything is natural except the supernatural. For a molecular biologist it is "against reality" and the very essence of science to regard genetic engineering as problematic in itself. The difference between a hen that has two wings, two legs, a head with a beak, living in an hierarchy with a rooster on top, and a "hen" that is just a growing and egg-laying "meatball", without legs, head and wings, is totally without significance to *science*. *In vivo* the argument is then: "of course, science is not a part of the ethical discourse. That is exactly the point. It is the ethicist (or the lay-person) that is granted that role". The response to this is to say that we think like this because we already take the ontology for granted. It is the ontology (that we say should not impinge on ethics) that instructs us to say "it is the ethicist (or the lay-person) that is granted that role". That is, it is the scientist who instructs ethics. Taking the ontology for granted has a double edge: you cannot ascribe any naturalness or intrinsic value without being constructivist in some sense and you cannot engage in ontology to defend views that could ascribe naturalness or intrinsic value to something, without being accused of committing "a naturalistic fallacy".

The normative feature of the metaphysics of natural science which describes nature as morally irrelevant (sic) is partly based in some epistemological values. What is here called "neutral knowledge" (and the authority that accompanies such knowledge) is only ideally realised through a strict division between subject and object. This is predicated on the presupposition that the object is passive. An "active object" would blur the whole "agreement" between subject and object.[3] If the object were active (in a way that made the observation relative to it), it would be less obvious to call the knowledge objective or neutral in the prescribed sense.

The epistemological authority of science is thus grounded in the same passivist view on nature.[4] Epistemology is ontologically determined and

[3] Cf. the paradigmatic example of Heisenberg's uncertainty principle.

[4] Within the same argument: the authority of social sciences are placed lower and the natural sciences higher or *vice versa* in the hierarchy of sciences, since the social scientist has to interpret and understand an "object that is active" in the strong mode of interpreting itself. (Double hermeneutics: cf. Heidegger on "Existenz" in *Sein und Zeit* at §25 and §9).

therefore also on this account normative. The normativity of epistemology is widely acknowledged but the normal reaction to it is to take for granted the assumption that we can never escape our epistemological boundaries. A critique of the scientific enterprise therefore often ends in a relativistic view on knowledge — leaving the passivist view on nature beyond the purview of criticism. We then fail to understand that this relativism is based in the passivist view on nature because it only gains purchase in the light of the "knowledge-requirements" of the passivist view on nature. Relativism and constructivism depend on the passivist ontology and this configuration, in turn, underpins expessivism and similar positions within ethics.[5]

Other (realist) ontologies could under a certain reasoning lead us into different lines of analysis both when it comes to the role of ethics and to epistemology. Disposition ontology exemplifies such an ontology (See e.g. Crane 1996; Mumford 1998, 2004; Molnar 2003; Martin 2007; Bird 2007). Here however, I will not elaborate that but rather try to deepen the points I have already made by taking a look at how the view worked out here would interpret the public debates that are going on in the field between ontology and ethics.

3. The debate in praxis

a. Post-normal Science

Silvio Funtowicz and Jerome Ravetz, coiners of the notion "post-normal science", also suggest that the strong is/ought distinction should be revised. Their argument however is made in terms of the external consequences that science through technology has co-created.

Their argument may be reproduced in the following way. As long as "science seemed overwhelmingly and essentially beneficial" it could also get away with very loosely founded conceptions of its role as "providing our ethical minds with facts" (Ravetz and Funtowicz 1999, p. 642). But now science has become a co-creator of a post-normal situation — a situation resulting in vast numbers of crises and basic uncertainty — it cannot still claim neutrality (Funtowicz and Ravetz 1992). Neutrality today would

[5] This is of course not what the expressivist would say, since he already takes it for granted that his positions have nothing to do with metaphysics.

mean something broader and something that also admits some space for ethical considerations.

Their strategy is to bring (particular parts of) science into a process of extended peer-reviews where "all possible views" (that is, the views of all possible stakeholders), should be represented and where quality "rather than abstract truth" is the governing value (Funtowicz and Ravetz 1994b).

I do agree with their description of our post-normal situation, but I don't agree on their solution to it. The disagreement is most easily illustrated by highlighting the communication aspect of their solution. As I believe I have shown, the passivist view on nature can not, by definition, ascribe any independent value to nature. What I now argue is that Funtowicz' and Ravetz' strategy ends up allowing only an ethical debate that has utilitarian or consequentialist frames. It is not possible to communicate other positions within a passivist regime and it is my contention that this regime remains a given of their approach. For this (ontological) problem it does not help to give endless descriptions of nature's complexity (Funtowicz and Ravetz 1994), since that only amounts to saying that "the consequences are complex". What could help is rather a theoretical (ontological) foundation that allows a possible space for ascribing a nature to nature. The following example will maybe demonstrate why this seemingly "anachronistic" move is needed.

Norman Levitt, a professor of mathematics, suggests that the European resistance to genetically modified organisms (GMOs) can be deconstructed through looking at "their particular cultural *dogmas of purity and danger*". Levitt uses the anthropologist Mary Douglas to analyse the situation.[6] In my view the disagreement between the "Americans" and the "Europeans" could have been a plain disagreement between two different "cultural interpretations", but in Levitt's view it is obviously not. As a scientist Levitt supposes himself to be in a situation that enables him to disavow the "European arguments" against GMO just by calling attention to the fact that "this is a cultural interpretation", thus implying that there exists a "non-cultural interpretation" of genetic engineering (GE). This seems a possible conjecture, but only on the grounds of the

[6] Levitt writes: "Why should this [GMO] have promoted so much concern? (...) Mary Douglas in her book *Purity and Danger*, puts forward the idea that cultures assume that there are "natural" categories, the transgression of which will bring about retribution. This obviously underlies much of the uneasiness concerning GM foods." (Stangroom 2005, p. 148).

molecular biological passivist view on genes: there is no nature of nature, and therefore nothing that could be "unclean" or "pure". This means that every critique of GE as such would be "cultured", while the absence of critique would not be. In this case, as viewed by Levitt, we would be able to say that "the Europeans" have interpreted the technology while "the Americans" have managed to avoid that. To explain why the scientist *speaking as a scientist* can propose such a conjecture, we have to recognize the significance of the passivist view of natural science.

b. "Risk-Ethics"

Anyone who takes a closer look at the GE debate will find it somewhat impenetrable. It looks as if the positions taken do not communicate well with each other. The "un-reflected" lay-person's concept of nature, employing the concept of naturalness, is "pedagogically rejected" by the scientist (Meyer and Sandøe 2001) but is defended by the "environmentalist" with the argument of intrinsic value (Verhoog 2003). The argument of intrinsic value, on the other hand has, as we have seen, no basis in the standard ontology of science and therefore no basis in "reality" either. The argument from naturalness is, for the reasons given, an argument that does not at all communicate with the scientist and the entrepreneurs of technology. The issue is therefore forced into a narrower discourse that *seems to be* the last common ground for both proponents and opponents, namely "risk-evaluations" (Wynne 2001). The debate about GMOs and gene-technology in general is about risks. This is a debate that apparently conforms to the scientist's passivist worldview.

In this debate discussing risks amounts to the question: does the biology *work as planned*? Under the assumed account it would be sufficient to investigate the "biological functions" that are intended by the altered genetic modifications.

There is however a problem here. Does not the question "does it work?" need a reference that values specific kinds of nature, (as, for example, defining the function of a car requires definite purposes of the thing called a car)? What kind of ontology do we need to assume that nature has some function? If function and functional explanations are to be deployed one has to recognize the existence of organisms and the whole web of interconnections that has developed through the course of evolution — interconnections that belong to the organism. But then we would have to talk about "right and wrong nature": that is, some nature in nature; a nature that could work as some kind of measuring stick for the function of the GMOs we want to put on the market and in the field. What would be at risk at all if there were no state or process (that was independent of humans) that could be disturbed or destroyed; if every particular were independent from the other in a way that made their properties *compossible* with anything else without *any* further "causal consequences"? However, to talk about a nature in nature in this way would be to contradict the scientific ontology that is used to ridicule the lay-person when he or she tries to make a case for naturalness in the GMO debate. The least ridiculous thing for a scientist to do would then be not to take part, even in the risk debate. As long as the concept of naturalness as such is not recognized as a viable concept in the debate there should be no scientific panels approving or disapproving GMO products. That would also be the fair and right thing to do according to the strong is/ought distinction that is employed elsewhere, when suitable.

Could the following describe what is happening with scientists in the GMO debate? Bio-molecular science has already "neutrally" stated that the change in genetic dispositions is as *insignificant* as moving a grain of sand from one place to another (cf. the substantial equivalence principle). At the same time "the scientist" *somehow* recognizes that there is an ethical aspect hidden in the area under discussion. However, blinded by the success in narrowing down the debate to a "risk question", she overlooks the fact that a risk judgement necessarily presupposes an evaluation of what is natural or at least functionally adequate — concepts that totally transcend the scope of molecular biology from whence the *insignificant view* originates.

More generally we could say that "the scientist" starts out with a "normative non-normativity". This normativity, which is built into his passivist ontology, is invisible to him or her, and he or she therefore ends up making "neutral" evaluations on a basis that totally contradicts the official ontology (*passivism*) of natural science.

In this situation the ethicist seems to be an obedient "placeholder" who never comes upon the core of the question and ends up where he starts: in his "eunuch-ethics". And this eunuch-ethics will follow the peer-reviews of post-normal science no matter how wide or quality-oriented this discourse will ever become.

"The scientist" will, of the measures proposed by the insight of post-normal science, interpret every extended peer-review as a category mistake, a substitution of truth with evaluations hampered by political correctness. I believe this will be the case with every suggestion that does not try to clear up the ontological problems that found the whole situation.

There is obviously a further way to argue for the view put forward here by showing directly how an alternative ontology would make a difference. For various reasons I have not given that priority here. But hopefully I have by now established a willingness to see that there is a present ontology of science but that this is not the only one and *thereby* neutral as e.g. Levitt typically assumes.

We often cling to the opinion that the present (passivist) ontology is neutral because we are unable to see any alternative other than the crude teleological one which for most people seems even more untenable. Other alternatives might be too new-age inspired and are often (willingly) not in compliance with basic scientific methods (e.g. Sheldrake, 1990). A leap hole for the many who disagree with the passivist ontology seems to be "complexity theories" such as chaos theories, probability theories and emergentism. These theories appear however not to be ontological in the sense that I try to convey here. They look more like extrapolated science. I believe that ontological arguments are needed against the passivist view, since that is what grounds its strength. I can show that the connections in the world are complex, but a Humean can show that they are totally contingent and an Armstrongian can show that they are totally compossible. I have pointed to the disposition ontology as a way to go. On this occasion I do not pursue the arguments for that view.

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Induction and Primitive Recursion in a Resource Conscious Logic — With a New Suggestion of How to Assign a Measure of Complexity to Primitive Recursive Functions

Uwe Petersen

Abstract. In [22], I presented a general approach to the definition of primitive recursive functions on the basis of a higher order logic without contraction employing a new kind of infinitary inference, the **Z**-inferences. The present paper is essentially a rewriting of this approach based on fixed-point constructions for the primitive recursive functions and a particular concern for the number of **Z**-inferences involved in proving results such as the recursion equations of primitive recursive functions and their totality.

1. Introduction

> Ever since the recursive functions have been identified there was a challenge to measure their inherent computational complexity, or in Kleene's words ⟦[15]⟧ to "classify the recursive functions into a hierarchy, according to some general principle".[1]

The present paper can be seen as a somewhat outlandish attempt to contribute to the problem of classifying primitive recursive functions.[2] It is based on a treatment of induction within a type free logic where "type free logic" is here used in the sense of [1] to mean that a logic "does not only possess formally the property of freedom of types, but beyond that an unrestricted axiom of comprehension."[3] Since the meaning of "unrestricted axiom of comprehension" may leave room for interpretation

[1] [25], p. 534.

[2] "Outlandish" in the sense that the author's primary research interest is dialectic in the Hegelean tradition and the ideas underlying the present contribution come out of that framework.

[3] On p. 3; my translation.

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(regarding the implication involved) I wish to specify that I require rules of the form

$$\frac{\mathfrak{F}[s]}{s \in \lambda x\, \mathfrak{F}[x]} \qquad \text{and} \qquad \frac{s \in \lambda x\, \mathfrak{F}[x]}{\mathfrak{F}[s]}$$

to be at least admissible, if not derivable.[4]

The basic idea for the type free logic employed here is to sacrifice contraction in exchange for unrestricted abstraction. Logic without contraction has this endearing feature to its credit that it allows a cut elimination proof which does not make recourse to the complexity of the cut formula. This is what makes logic without contraction such an ideal candidate for an underlying logic of a type free theory: the unpredictable way in which abstraction may change the complexity of the cut formula is irrelevant to a proof of cut elimination. It is also what has recently made it attractive to theoretical computer scientists in their quest for a "logic of polytime".[5]

But higher order logic without contraction is also a promising basis for a logical foundation in the style envisaged by Frege, no longer marred by inconsistencies. In other words, it is possible to take up again the reductionist approach in the foundations of mathematics after it has been cleared of the danger of antinomies stemming from unrestricted abstraction.[6]

The traditional way (Dedekind) of defining primitive recursive functions in a higher order logic follows the schema of induction. In the case of addition it commonly looks somewhat like this:

$$\begin{aligned} (1.1)\quad \mathcal{A} :\equiv \lambda x_1 x_2 x_3 \bigwedge y (\langle\langle x_1, 0\rangle, x_1\rangle \in y \wedge \\ \bigwedge z_1 \bigwedge z_2 (\langle\langle x_1, z_1\rangle, z_2\rangle \rightarrow \langle\langle x_1, z_1'\rangle, z_2'\rangle) \rightarrow \langle\langle x_1, x_2\rangle, x_3\rangle \in y) \end{aligned}$$

with $s + t :\equiv \mathcal{A}[\![s, t]\!]$. I have taken this road in [21], sections 137a & 137b and repeated in [22], section 5.

[4] Type free logics of the kind presented in [2] and [6] are not "type-free" in this (strong) sense.

[5] Cf. [8].

[6] To be sure, this is not the only reduction that looks promising. Having gone through the experience of running into antinomies, higher logic now shares with metaphysics what Kant called the *Dialectic of Pure Reason* and my hope would be that metaphysics in turn can profit a bit from the methods that have been developed in foundational studies of mathematics and logic.

A certain "impredicativity" comes in here through the bound variable y being ruled by generalization. Employing fixed points this can be achieved "cheaper". Addition is declared as the fixed point $\mathcal{A}$ satisfying:

$$(1.2)\quad \mathcal{A} = \lambda x_1 x_2 x_3 ((x_2 = 0 \wedge x_3 = x_1) \vee \bigvee y_1 \bigvee y_2 \bigvee y_3 (y_1 = x_1 \sqcap y_2' = x_2 \sqcap y_3' = x_3 \sqcap \langle\langle y_1, y_2\rangle, y_3\rangle \in \mathcal{A}) .$$

Here the recursive character (calling itself up) comes from an unabashed application of the fixed point property.

Very little is actually needed on top of unrestricted abstraction to be able to prove a general fixed point property for term-forms $\mathfrak{F}$

$$(1.3)\qquad f = \lambda x\, \mathfrak{F}[f, x]$$

and what is needed is not lost by giving up contraction.[7]

With 1.3 at hand, terms for primitive recursive functions can be introduced according to 1.2 rather than the more traditional "second order" style indicated in 1.1 and thereby save a little bit on inductions. But when it comes to proving recursion equations and totality, some form of induction is indispensable. If one is only interested in a numerical representation, meta-theoretical inductions are sufficient. But for a proof of recursion equations with proper variables, induction on the formal level is required.

Due to the lack of contractions, however, special methods have to be introduced to achieve what can usually by accomplished by induction. Since the consistency of higher order logic without contraction is provable by ordinary induction, it will be clear that induction cannot be provable on the formal level. Actually, induction in its classical form can be shown to be incompatible with $\mathbf{L^iD}_\lambda$.[8]

In a higher order logic, induction is provided by a term of the form

$$\lambda x \bigwedge y (\bigwedge z (z \in y \to z' \in y) \to (0 \in y \to x \in y)) .$$

[7] The first clear statement of this (for the case of a type free logic) seems to be in [8], p. 173, proposition 4. Note, however, that Girard reserves the symbol $=$ for identity (for which I use $\equiv$) which is why his formulation looks slightly different. Cf. also [26], theorem 2, [20], p. 382, [5], p. 357, [19], theorem 10. In [20], lemma 7.2, I employed a notion of *application* (cf. definitions 2.4 below) which resulted in a more roundabout way of proving the fixed point property. Employing the notion of *co-domain of a relation* instead simplifies matters (cf. also [24], p. 122).

[8] Cf. [24], section 10.

This, however, no longer works without contraction: all one gets is that 0 and $0'$ fall under that term. What is lacking is the possibility of repeating the "induction step"

$$\bigwedge z\,(z \in t \to z' \in t)$$

ad libitum without having to "pay extra for it". In classical logic a wff includes a "use-as-often-as-you-like" license, and that by virtue of the axioms for implication.[9]

Desirable would be a way of expressing that assumptions can be used more often that once, but that this has to be accounted for.[10] In classical logic assumptions can always be used more often than just once, but one is not required to keep track of multiple uses.

Induction on the basis of classical logic is cheating: the problem of articulating "how many" doesn't arise thanks to contraction. Frege's analysis was more focused on the issue of a number being an equivalence class, than on the problem of how one can establish that 3, for instance, is a natural number without using the step (adding 1) three times. In a logic without contraction the notion of number is strongly tied to being able to repeat a particular operation, *viz.*, the application of the successor operation.

It is in these special methods that a kind of complexity comes in which is completely absent from a classical approach: keeping track of assumptions (resource consciousness).

Now I cannot claim to feel at home in the area of computational complexity nor do I feel confident to enter the discussion. However, engaging with the problem of recovering induction and recursion in a contraction free logic with unrestricted abstraction, I found myself placed in the neighborhood of questions concerning the possibility of classifying the recursive functions into a hierarchy. But, as I indicated in the introduction to this paper, my suggestion is a strange ("maverick") one, at least from a classical perspective: it is intimately linked to the way I introduced induction in [20] and employed it in [22]. This, in turn, cannot be separated from my way of treating infinity, *viz.*, through the introduction of **Z**-inferences. It

[9] That there is a problem with implication in a type free logic has long been observed. Paradoxes of the kind usually attributed to Curry require a restriction of implication which makes it impossible to obtain from the above formulation what is required for implication.

[10] This is what makes the logic "resource conscious": recycling of assumptions comes at a cost.

is the number of **Z**-inferences necessary to prove a result that will provide a measure for complexity.[11]

One last word before I close this introduction. The work presented here is of extremely basic nature and presenting deductions may well be regarded as a trivial exercise. But the point is to track down inferences that account for certain "totalizations", as I am inclined to call them, which I hope can provide a measure of complexity. In the course of trying to do this, I have made so many mistakes, mostly by being caught in a classical way of thinking, that I decided it would be better, at least for me, to write down deductions in virtually full length. This will enable those who are prepared to take the trouble of ploughing through details to see where I might have gone wrong and whether it will invalidate my project.

2. Basic notions

In this section I mainly repeat definitions and provide a few basic results that have been established in [20] and, above all, in [21].[12]

Definitions 2.1. (1) Primitive symbols:

(1.1) symbols for free and bound *variables*: a, b, c, and x, y, z, also with index numbers;

(1.2) the *constants* λ, $\in$, $\bigwedge$, $\to$, and $\square$.

(2) The language $\mathcal{L}$ is defined accordingly.

Remark 2.2. This is not the most economical choice of primitive symbols, but rather an attempt at making more accessible considerations regarding the notion of Z-specific wffs introduced in [20], p. 388.

Definitions 2.3. (1) Initial sequents: $A \Rightarrow A$.
(2) Structural rules:

$$\text{Weakening}: \quad \frac{\Gamma \Rightarrow C}{A, \Gamma \Rightarrow C}\,, \qquad \text{Exchange}: \quad \frac{A, \Gamma \Rightarrow \mathfrak{F}[s]}{\Gamma \Rightarrow s \in \lambda x\, \mathfrak{F}[x]}\,,$$

[11] It should be clear that **Z**-inferences are not needed in order to prove numerical results like $3 + 2 = 5$, for instance.

[12] Some of the following definitions may slightly differ from the ones I have given elsewhere. It should be clear, however, that they are logically equivalent to the ones given there, if not explicitly stated otherwise.

$$\text{Cut}: \quad \frac{\Gamma \Rightarrow A \qquad A, \Pi \Rightarrow}{\Gamma, \Pi \Rightarrow B}\ .$$

(3) Operational rules.
(3.1) Rules for $\in$:

$$\text{left}: \quad \frac{\mathfrak{F}[s], \Gamma \Rightarrow C}{s \in \lambda x\, \mathfrak{F}[x], \Gamma \Rightarrow C}\ ; \qquad \text{right}: \quad \frac{A, \Gamma \Rightarrow \mathfrak{F}[s]}{\Gamma \Rightarrow s \in \lambda x\, \mathfrak{F}[x]}\ .$$

(3.2) Rules for $\bigwedge$:

$$\text{left}: \quad \frac{\mathfrak{F}[s], \Gamma \Rightarrow C}{\bigwedge y\, \mathfrak{F}[y], \Gamma \Rightarrow C}\ ; \qquad \text{right}: \quad \frac{\Gamma \Rightarrow \mathfrak{F}[a]}{\Gamma \Rightarrow \bigwedge y\, \mathfrak{F}[y]}\ .$$

with the usual condition on the *eigenvariable* a.
(3.3) Rules for $\rightarrow$:

$$\text{left}: \quad \frac{\Gamma \Rightarrow A \qquad B, \Pi \Rightarrow C}{A \rightarrow B, \Gamma, \Pi \Rightarrow C}\ ; \qquad \text{right}: \quad \frac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A \rightarrow B}\ .$$

(3.4) Rules for $\square$:

$$\text{left}: \quad \frac{A, B, \Gamma \Rightarrow C}{A \,\square\, B, \Gamma \Rightarrow C}\ ; \qquad \text{right}: \quad \frac{\Gamma \Rightarrow A \qquad \Pi \Rightarrow B}{\Gamma, \Pi \Rightarrow A \,\square\, B}\ .$$

(4) The formalized theory $\mathbf{L^iD}_\lambda$ is defined as the language $\mathcal{L}$ with the foregoing initial sequents and rules of inference.

Definitions 2.4. The following is a list of defined constants:[13]

$$\begin{aligned}
s \subseteq t \ &:\equiv \bigwedge x\,(x \in s \rightarrow x \in t)\,; \\
\mathcal{V} \ &:\equiv \lambda x\,(x \subseteq x)\,; \\
\bot \ &:\equiv \ \bigwedge x\,(\mathcal{V} \subseteq x)\,; \\
\neg A \ &:\equiv \ A \rightarrow \bot\,; \\
\emptyset \ &:\equiv \ \lambda \bot\,; \\
\top \ &:\equiv \ \neg\bot\,; \\
\bigvee x\, \mathfrak{F}[x] \ &:\equiv \bigwedge y\,(\bigwedge x\,(\mathfrak{F}[x] \rightarrow \lambda\top \subseteq y) \rightarrow \lambda\top \subseteq y) \quad \text{(existence)}\,; \\
s \equiv t \ &:\equiv \bigwedge y\,(s \in y \rightarrow t \in y) \qquad \text{(identity)}\,;
\end{aligned}$$

[13] This list is in large parts identical to that in [20], p. 66 f. It is provided here for the sake of convenience.

$$\begin{aligned}
A \Box B &:\equiv \bigwedge x\,((A \to (B \to \lambda\top \subseteq x)) \to \lambda\top \subseteq x)\,;\\
A \wedge B &:\equiv \bigwedge x\,(\bigwedge y\,(\lambda A \in y \to (\lambda B \in y \to x \in y)) \to \lambda\top \subseteq x)\,;\\
A \vee B &:\equiv \bigwedge y\,((A \to \lambda\top \subseteq y) \wedge (B \to \lambda\top \subseteq y) \to \lambda\top \subseteq x)\,;\\
\{\!\{s\}\!\} &:\equiv \lambda x\,(s \equiv x) \qquad \text{(“exclusive” singleton)}\,;\\
\{\!\{s,t\}\!\} &:\equiv \lambda x\,(x \equiv s \vee x \equiv t) \qquad \text{(“exclusive” pairing)}\,;\\
A \leftrightarrow B &:\equiv (A \to B) \wedge (B \to A)\,;\\
s = t &:\equiv \bigwedge x\,(x \in s \leftrightarrow x \in t) \qquad \text{(equality)}\,;\\
\{s\} &:\equiv \lambda x\,(x = s) \qquad \text{(“inclusive” singleton)}\,;\\
\{s,t\} &:\equiv \lambda x\,(x = s \vee x = t) \qquad \text{(“inclusive” pairing)}\,;\\
\langle s,t\rangle &:\equiv \{\{\{s\},\emptyset\},\{\{t\}\}\} \qquad \text{(“inclusive” ordered pair)}\,;\\
\langle\!\langle s,t\rangle\!\rangle &:\equiv \{\!\{\{\!\{\{\!\{s\}\!\},\emptyset\}\!\},\{\!\{\{\!\{t\}\!\}\}\!\}\}\!\} \qquad \text{(“exclusive” ordered pair)}\,;\\
\lambda xy\,\mathfrak{F}[x,y] &:\equiv \lambda z \bigvee x \bigvee y\,(z \equiv \langle\!\langle x,y\rangle\!\rangle \Box \mathfrak{F}[x,y]) \qquad \text{(dyadic abstract)}\,;\\
s \cup t &:\equiv \lambda x\,(x \in s \vee x \in t)\,;\\
s \sqcap t &:\equiv \lambda x\,(x \in s \Box x \in t)\,;\\
st &:\equiv \lambda x\,(\langle\!\langle t,x\rangle\!\rangle \in s) \qquad \text{(co-domain of a relation)}\,;\\
s[\![t]\!] &:\equiv \lambda x \bigwedge y\,(\langle\!\langle t,y\rangle\!\rangle \in s \to x \in y) \qquad \text{(application)}\,.
\end{aligned}$$

I repeat a few notational conventions from [21].

CONVENTIONS 2.5. (1) $[A]^{\cdot 2} :\equiv A \Box A$.
(2) $[A/s] :\equiv \lambda A \in s$.
(3) $\mathrm{k}\,[A]$ is inductively defined as follows:
(3.1) $1\,[A] :\equiv A$;
(3.2) $\mathrm{k}'\,[A] :\equiv A, \mathrm{k}[A]$.

Before being able to express induction over numbers, I need a way of expressing that an assumption may be used a certain number of times. The next definition provides some basic ingredients.

DEFINITIONS 2.6. (1) $I :\equiv \lambda x(x = \mathcal{V})$, *i.e.*, $\{\mathcal{V}\}$.
(2) $s^I :\equiv \lambda x(x \in s \Box x \in I)$, *i.e.*, $s \sqcap I$.

Definitions 2.6 allow us to express (and prove) simple properties such as $[A/I^I] \leftrightarrow A \,\Box\, A$.

PROPOSITION 2.7. *The following is* $\mathbf{L^i D}_\lambda$*-deducible:*

(2.7i) $[A/I] \leftrightarrow A$;

(2.7ii) $[A/s^I] \leftrightarrow [A/s] \,\Box\, A$;

(2.7iii) $[A/s \sqcap t] \Rightarrow [A/s] \,\Box\, [A/t]$;

(2.7iv) $[A/s], [A/t] \Rightarrow [A/s \sqcap t]$;

(2.7v) $s = t, [A/s] \Rightarrow [A/t]$.

Proof. Cf. [21], p. 1792. QED

DEFINITION 2.8. $s \sqcap^{\mathrm{k}} t$ is defined inductively as follows:

(1) $s \sqcap^1 t :\equiv s \sqcap t$;

(2) $s \sqcap^{\mathrm{k}'} t :\equiv (s \sqcap^{\mathrm{k}} t) \sqcap t$.

PROPOSITION 2.9. *The following is* $\mathbf{L^i D}_\lambda$*-deducible:*

(2.9i) $\Rightarrow (s \sqcap t)^{II} = (s^I \sqcap t^I)$;

(2.9ii) $\Rightarrow ((s \sqcap t) \sqcap r)^{III} = (s^I \sqcap t^I) \sqcap r^I$;

(2.9iii) $\Rightarrow (s \sqcap^{\mathrm{k}} r) \sqcap (t \sqcap^{\mathrm{k}} r) = (s \sqcap t) \sqcap^{2\mathrm{k}} r$.

Proof. These are straightforward consequences of the associativity and commutativity of $\sqcap$; left to the reader. QED

DEFINITIONS 2.10. (1) The (intuitive) set $\mathbf{\Psi}$ is defined inductively as follows:

(1.1) I is an element of $\mathbf{\Psi}$;

(1.2) If s is an element of $\mathbf{\Psi}$, then so is s^I.

(2) If $\mathrm{m} \in \mathbb{N}\backslash\{0\}$, then its *corresponding* $\mathbf{\Psi}$*-element* is defined inductively as follows:

(2.1) I is the *corresponding* $\mathbf{\Psi}$*-element* of 1.

(2.1) If $\tilde{n}$ is the corresponding $\mathbf{\Psi}$-element of n, the $\tilde{n}^I$ is the *corresponding* $\mathbf{\Psi}$*-element* of n'.

REMARK 2.11. In the appendix, section 12 at the end of this paper, this correspondence between natural numbers > 0 and elements of $\mathbf{\Psi}$ will be established on the formal level.

DEFINITIONS 2.12. (1) $\breve{\gamma}[A] :\equiv \lambda x((x \in x) \sqcap A) \in \lambda x((x \in x) \sqcap A)$.[14]
(2) $\mathbf{Z} :\equiv \lambda x \bigwedge y(\breve{\gamma}[\bigwedge z(z \in y \rightarrow z^I \in y)] \rightarrow (I \in y \rightarrow x \in y))$.

I list a few properties of $\breve{\gamma}$.

PROPOSITION 2.13. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(2.13i) $\breve{\gamma}[A] \Rightarrow \breve{\gamma}[A] \sqcap A$;

(2.13ii) $\breve{\gamma}[A] \Rightarrow A$;

(2.13iii) $\breve{\gamma}[A \rightarrow B], \breve{\gamma}[A] \Rightarrow B$;

(2.13iv) $\breve{\gamma}[A] \rightarrow (A \rightarrow B), \breve{\gamma}[A] \Rightarrow B$.

Proof. Cf. [21], p. 1804. QED

PROPOSITION 2.14. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(2.14i) $\Rightarrow I \in \mathbf{Z}$;

(2.14ii) $s \in \mathbf{Z} \Rightarrow s^I \in \mathbf{Z}$;

(2.14iii) $s \in \mathbf{Z} \Rightarrow (s \sqcap^{\mathrm{k}} I) \in \mathbf{Z}$.

Proof. As regards 2.14i and 2.14ii, cf. [21], p. 1806. 2.14iii is obtained by a straightforward induction on k which is left to the reader. QED

DEFINITIONS 2.15.
(1) $\breve{\mathbf{\Pi}}^\circ :\equiv \lambda x(x \in \mathbf{Z} \sqcap \bigwedge y([I \in y \wedge \bigwedge z(z \in y \rightarrow z^I \in y)/x] \rightarrow x \in y))$.
(2) $\Box A :\equiv \bigwedge x(x \in \breve{\mathbf{\Pi}}^\circ \sqcap [A/x])$.
(3) $A \supset B :\equiv \bigvee x(x \in \breve{\mathbf{\Pi}}^\circ \sqcap ([A/x] \rightarrow B))$.
(4) $\breve{\mathbf{\Pi}} :\equiv \lambda x \bigvee y(y \in \breve{\mathbf{\Pi}}^\circ \sqcap y = x)$.[15]

PROPOSITION 2.16. *Inferences according to the following schema are* $\mathbf{L^iD}_\lambda$*-derivable:*

$$\frac{B, A, \Gamma \Rightarrow C}{A \supset B, \Gamma \Rightarrow A \supset C} \; .$$

[14] This is just an explicit fixed point construction.

[15] Now that *is* different from the one in [20], p. 398, but hopefully no reason for concern. Cf. also footnote 22 below.

Proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{a\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow a^{I}\in\breve{\mathbf{\Pi}}^{\circ}\qquad \dfrac{\dfrac{\dfrac{\dfrac{[A/a]\Rightarrow[A/a]\qquad B,A,\mathit{\Gamma}\Rightarrow C}{[A/a]\to B,[A/a],A,\mathit{\Gamma}\Rightarrow C}}{[A/a]\to B,[A/a^{I}],\mathit{\Gamma}\Rightarrow C}}{[A/a]\to B,\mathit{\Gamma}\Rightarrow[A/a^{I}]\to C}}{a\in\breve{\mathbf{\Pi}}^{\circ},[A/a]\to B,\mathit{\Gamma}\Rightarrow a^{I}\in\breve{\mathbf{\Pi}}^{\circ}\sqcap([A/a^{I}]\to C)}}{a\in\breve{\mathbf{\Pi}}^{\circ}\sqcap([A/a]\to B),\mathit{\Gamma}\Rightarrow a^{I}\in\breve{\mathbf{\Pi}}^{\circ}\sqcap([A/a^{I}]\to C)}}{a\in\breve{\mathbf{\Pi}}^{\circ}\sqcap([A/a]\to B),\mathit{\Gamma}\Rightarrow\bigvee x(x\in\breve{\mathbf{\Pi}}^{\circ}\sqcap([A/x]\to C))}}{\bigvee x(x\in\breve{\mathbf{\Pi}}^{\circ}\sqcap([A/x]\to B)),\mathit{\Gamma}\Rightarrow\bigvee x(x\in\breve{\mathbf{\Pi}}^{\circ}\sqcap([A/x]\to C))}}{A\supset B,\mathit{\Gamma}\Rightarrow A\supset C}\;.$$

QED

REMARK 2.17. The point of the foregoing result: the formulation of $\mathbf{N}^{\circ}$ in definition 4.4 below in terms of the weak implication $\supset$ does not make additional deductive power necessary when it comes to establishing $s\in\mathbf{N}^{\circ}\Rightarrow s'\in\mathbf{N}^{\circ}$ (4.6ii below).

PROPOSITION 2.18. *The following is* $\mathbf{L^{i}D}_{\lambda}$*-deducible:*

(2.18i) $\Rightarrow I\in\breve{\mathbf{\Pi}}^{\circ}$;

(2.18ii) $s\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow s^{I}\in\breve{\mathbf{\Pi}}^{\circ}$;

(2.18iii) $s\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow(s\sqcap^{\mathrm{k}}I)\in\breve{\mathbf{\Pi}}^{\circ}$.

Proof. As regards 2.18i and 2.18ii cf. 134.3iii and 134.3iv in [20], p. 1825. It must be clear though that these are indeed $\mathbf{L^{i}D}_{\lambda}$-deducible, *i.e.*, no **Z**-inference required.[16] 2.18iii as for 2.14iii. QED

3. Z-inferences

As it stands, $\breve{\mathbf{\Pi}}^{\circ}$ doesn't offer much of an advantage as against **Z**. This is now going to change with the introduction of **Z**-inferences.

[16] In [20], they were listed as $\mathbf{L^{i}D}_{\lambda}^{\mathrm{Z}}$-deducible.

DEFINITIONS 3.1. (1) An inference according to the schema

$$\frac{\Gamma \Rightarrow s \in \mathbf{Z} \qquad \Rightarrow A}{\Gamma \Rightarrow [A/s]}$$

is called a **Z**-inference.[17]
(2) The formalized theory $\mathbf{L^iD}_\lambda^{\mathrm{Z}}$ is obtained from $\mathbf{L^iD}_\lambda$ by adding all **Z**-inferences.

In what follows I shall mostly consider "throttled" versions of $\mathbf{L^iD}_\lambda^{\mathrm{Z}}$.

DEFINITION 3.2. $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction \mathrm{n}}$ is defined as $\mathbf{L^iD}_\lambda^{\mathrm{Z}}$ with the restriction that a $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction \mathrm{n}}$-deduction can contain at most n **Z**-inferences.

PROPOSITION 3.3. *Inferences according to the following schemata are* $\mathbf{L^iD}_\lambda^{\mathrm{Z}}$*-derivable with an increase of the* **Z***-grade indicated on the right:*

(3.3i)
$$\frac{\Gamma \Rightarrow \mathfrak{F}[I] \qquad \mathfrak{F}[a] \Rightarrow \mathfrak{F}[a^I]}{s \in \breve{\mathbf{\Pi}}^\circ, \Gamma \Rightarrow \mathfrak{F}[s]}\,{\scriptstyle +1}\,;$$

(3.3ii)
$$\frac{\Gamma, \mathfrak{A}[I] \Rightarrow \mathfrak{B}[I] \qquad \mathfrak{A}[a] \rightarrow \mathfrak{B}[a], \mathfrak{A}[a^I] \Rightarrow \mathfrak{B}[a^I]}{s \in \breve{\mathbf{\Pi}}^\circ, \Gamma, \mathfrak{A}[s] \Rightarrow \mathfrak{B}[s]}\,{\scriptstyle +1}\,;$$

(3.3iii)
$$\frac{\Gamma \Rightarrow \mathfrak{F}[I] \qquad \mathfrak{F}[a] \Rightarrow \mathfrak{F}[a^I] \qquad \mathfrak{F}[s], \Gamma, \Pi \Rightarrow C}{s \in \breve{\mathbf{\Pi}}^\circ, \Gamma, \Pi \Rightarrow C}\,{\scriptstyle +1}\,;$$

(3.3iv)
$$\frac{s \in \breve{\mathbf{\Pi}}^\circ, s \in \breve{\mathbf{\Pi}}^\circ, \Gamma \Rightarrow C}{s \in \breve{\mathbf{\Pi}}^\circ, \Gamma \Rightarrow C}\,{\scriptstyle +1}\,;$$

(3.3v)
$$\frac{\Gamma \Rightarrow \mathfrak{F}[I] \qquad a \in \breve{\mathbf{\Pi}}^\circ, \mathfrak{F}[a] \Rightarrow \mathfrak{F}[a^I]}{s \in \breve{\mathbf{\Pi}}^\circ, \Gamma \Rightarrow \mathfrak{F}[s]}\,{\scriptstyle +2}\,.$$

[17] **Z**-inferences have been first introduced in [20], p. 392. I shall not here comment on the meta-theoretical side of these inferences but only refer curious readers to [21], section 119b, for a little bit of justification.

Proof. *Re* 3.3i.

$$\dfrac{\dfrac{\Rightarrow \mathfrak{F}[I]}{\Rightarrow I\in\lambda x\,\mathfrak{F}[x]} \qquad \dfrac{\dfrac{\dfrac{\mathfrak{F}[c]\Rightarrow\mathfrak{F}[c^I]}{c\in\lambda x\,\mathfrak{F}[x]\Rightarrow c^I\in\lambda x\,\mathfrak{F}[x]}}{\Rightarrow c\in\lambda x\,\mathfrak{F}[x]\rightarrow c^I\in\lambda x\,\mathfrak{F}[x]}}{\Rightarrow \bigwedge z(z\in\lambda x\,\mathfrak{F}[x]\rightarrow z^I\in\lambda x\,\mathfrak{F}[x])}}{\Rightarrow I\in\lambda x\,\mathfrak{F}[x]\wedge\bigwedge z(z\in\lambda x\,\mathfrak{F}[x]\rightarrow z^I\in\lambda x\,\mathfrak{F}[x])}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow I\in\lambda x\,\mathfrak{F}[x]\wedge\bigwedge z(z\in\lambda x\,\mathfrak{F}[x]\rightarrow z^I\in\lambda x\,\mathfrak{F}[x])}{a\in\mathbf{Z}\Rightarrow[I\in\lambda x\,\mathfrak{F}[x]\wedge\bigwedge z(z\in\lambda x\,\mathfrak{F}[x]\rightarrow z^I\in\lambda x\,\mathfrak{F}[x])/a]}{+1} \qquad \dfrac{\mathfrak{F}[s]\Rightarrow\mathfrak{F}[s]}{a\in\lambda x\,\mathfrak{F}[x]\Rightarrow\mathfrak{F}[s]}}{a\in\mathbf{Z},[I\in\lambda x\,\mathfrak{F}[x]\wedge\bigwedge z(z\in\lambda x\,\mathfrak{F}[x]\rightarrow z^I\in\lambda x\,\mathfrak{F}[x])/a]\rightarrow a\in\lambda x\,\mathfrak{F}[x]\Rightarrow\mathfrak{F}[s]}}{a\in\mathbf{Z},\bigwedge y([I\in y\wedge\bigwedge z(z\in y\rightarrow z^I\in y)/a]\rightarrow a\in y)\Rightarrow\mathfrak{F}[s]}}{a\in\mathbf{Z}\,\Box\bigwedge y([I\in y\wedge\bigwedge z(z\in y\rightarrow z^I\in y)/a]\rightarrow a\in y)\Rightarrow\mathfrak{F}[s]}}{s\in\breve{\mathbf{\Pi}}^{\circ},\Gamma\Rightarrow\mathfrak{F}[s]}\;.$$

Re 3.3ii. Essentially as for 3.3i. The point is to see that no cut (or inversion) is required. Let $\xi :\equiv \lambda x(\mathfrak{A}[x]\rightarrow\mathfrak{B}[x])$

$$\dfrac{\dfrac{\dfrac{\mathfrak{A}[I]\Rightarrow\mathfrak{B}[I]}{\Rightarrow\mathfrak{A}[I]\rightarrow\mathfrak{B}[I]}}{\Rightarrow I\in\xi} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\mathfrak{A}[a]\rightarrow[B/a],\mathfrak{A}[a^I]\Rightarrow[B/a^I]}{\mathfrak{A}[a]\rightarrow[B/a]\Rightarrow\mathfrak{A}[a^I]\rightarrow[B/a^I]}}{c\in\xi\Rightarrow c^I\in\xi}}{\Rightarrow c\in\xi\rightarrow c^I\in\xi}}{\Rightarrow\bigwedge z(z\in\xi\rightarrow z^I\in\xi)}}{\Rightarrow I\in\xi\wedge\bigwedge z(z\in\xi\rightarrow z^I\in\xi)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow I\in\xi\wedge\bigwedge z(z\in\xi\rightarrow z^I\in\xi)}{s\in\mathbf{Z}\Rightarrow[I\in\xi\wedge\bigwedge z(z\in\xi\rightarrow z^I\in\xi)/s]}{+1} \qquad \dfrac{\dfrac{\mathfrak{A}[s]\Rightarrow\mathfrak{A}[s]\qquad\mathfrak{B}[s]\Rightarrow\mathfrak{B}[s]}{\mathfrak{A}[s]\rightarrow\mathfrak{B}[s],\mathfrak{A}[s]\Rightarrow\mathfrak{B}[s]}}{s\in\xi,\mathfrak{A}[s]\Rightarrow\mathfrak{B}[s]}}{s\in\mathbf{Z},[I\in\xi\wedge\bigwedge z(z\in\xi\rightarrow z^I\in\xi)/s]\rightarrow s\in\xi),\mathfrak{A}[s]\Rightarrow\mathfrak{B}[s]}}{s\in\mathbf{Z},\bigwedge y([I\in y\wedge\bigwedge z(z\in y\rightarrow z^I\in y)/s]\rightarrow s\in y),\mathfrak{A}[s]\Rightarrow\mathfrak{B}[s]}}{s\in\mathbf{Z}\,\Box\bigwedge y([I\in y\wedge\bigwedge z(z\in y\rightarrow z^I\in y)/s]\rightarrow s\in y),\mathfrak{A}[s]\Rightarrow\mathfrak{B}[s]}}{s\in\breve{\mathbf{\Pi}}^{\circ},\mathfrak{A}[s]\Rightarrow\mathfrak{B}[s]}\;.$$

Re 3.3iii. Essentially as for 3.3ii; left to the reader. Cf. also 4.7ii below.

Re 3.3iv. This is a straightforward consequence of 3.3iii. Employ 2.18i and 2.18ii:

$$\dfrac{\dfrac{\Rightarrow I\in\breve{\mathbf{\Pi}}^{\circ} \qquad \Rightarrow I\in\breve{\mathbf{\Pi}}^{\circ}}{\Rightarrow I\in\breve{\mathbf{\Pi}}^{\circ}\,\square\, I\in\breve{\mathbf{\Pi}}^{\circ}} \qquad \dfrac{\dfrac{c\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow c^{I}\in\breve{\mathbf{\Pi}}^{\circ} \quad c\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow c^{I}\in\breve{\mathbf{\Pi}}^{\circ}}{c\in\breve{\mathbf{\Pi}}^{\circ}, c\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow c^{I}\in\breve{\mathbf{\Pi}}^{\circ}\,\square\, c^{I}\in\breve{\mathbf{\Pi}}^{\circ}}}{c\in\breve{\mathbf{\Pi}}^{\circ}\,\square\, c\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow c^{I}\in\breve{\mathbf{\Pi}}^{\circ}\,\square\, c^{I}\in\breve{\mathbf{\Pi}}^{\circ}} \qquad \dfrac{s\in\breve{\mathbf{\Pi}}^{\circ}, s\in\breve{\mathbf{\Pi}}^{\circ},\Gamma\Rightarrow C}{s\in\breve{\mathbf{\Pi}}^{\circ}\,\square\, s\in\breve{\mathbf{\Pi}}^{\circ},\Gamma\Rightarrow C}}{s\in\breve{\mathbf{\Pi}}^{\circ},\Gamma\Rightarrow C}\,{+1}.$$

Re 3.3v. Employ 3.3iv and 3.3i:

$$\dfrac{\dfrac{\Rightarrow I\in\breve{\mathbf{\Pi}}^{\circ} \qquad \Gamma\Rightarrow\mathfrak{F}[I]}{\Rightarrow I\in\breve{\mathbf{\Pi}}^{\circ}\,\square\,\mathfrak{F}[I]} \qquad \dfrac{\dfrac{\dfrac{a\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow a^{I}\in\breve{\mathbf{\Pi}}^{\circ} \qquad a\in\breve{\mathbf{\Pi}}^{\circ},\mathfrak{F}[a]\Rightarrow\mathfrak{F}[a^{I}]}{a\in\breve{\mathbf{\Pi}}^{\circ}, a\in\breve{\mathbf{\Pi}}^{\circ},\mathfrak{F}[a]\Rightarrow a\in\breve{\mathbf{\Pi}}^{\circ}\,\square\,\mathfrak{F}[a^{I}]}}{a\in\breve{\mathbf{\Pi}}^{\circ},\mathfrak{F}[a]\Rightarrow a\in\breve{\mathbf{\Pi}}^{\circ}\,\square\,\mathfrak{F}[a^{I}]}\,{+1}}{a\in\breve{\mathbf{\Pi}}^{\circ}\,\square\,\mathfrak{F}[a]\Rightarrow a\in\breve{\mathbf{\Pi}}^{\circ}\,\square\,\mathfrak{F}[a^{I}]}}{s\in\breve{\mathbf{\Pi}}^{\circ},\Gamma\Rightarrow\mathfrak{F}[s]}\,{+1}\,.$$

QED

REMARK 3.4. The reason for taking the detour *via* $\mathbf{Z}$ to get to $\breve{\mathbf{\Pi}}^{\circ}$ should become sufficiently clear by looking at the proof of 3.3i above. In view of its obvious similarity to induction, I shall occasionally refer to it as proto-induction.

PROPOSITION 3.5. *The following holds:*

(3.5i) $\quad \mathbf{L}^{\mathbf{i}}\mathbf{D}_{\lambda}\vdash\;\Rightarrow(I\sqcap^{\mathrm{k}} I)\in\breve{\mathbf{\Pi}}\,;$

(3.5ii) $\quad \mathbf{L}^{\mathbf{i}}\mathbf{D}_{\lambda}^{\mathsf{Z}\upharpoonright 1}\vdash\; s\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow(s\sqcap^{\mathrm{k}} s)\in\breve{\mathbf{\Pi}}\,.$

Proof. Re 3.5i. This is an immediate consequence of 2.18iii.
Re 3.5ii. This is a straightforward application of 3.3i employing 2.9i:

$$\dfrac{\Rightarrow(I\sqcap^{\mathrm{k}} I)\in\breve{\mathbf{\Pi}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{b\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow(b\sqcap^{2\mathrm{k}} I)\in\breve{\mathbf{\Pi}}^{\circ} \quad \dfrac{\Rightarrow(c\sqcap^{\mathrm{k}} c)\sqcap^{2\mathrm{k}} I=(c^{I}\sqcap^{\mathrm{k}} c^{I})}{b=(c\sqcap^{\mathrm{k}} c)\Rightarrow(b\sqcap^{2\mathrm{k}} I)=(c^{I}\sqcap^{\mathrm{k}} c^{I})}}{b\in\breve{\mathbf{\Pi}}^{\circ}, b=(c\sqcap^{\mathrm{k}} c)\Rightarrow(b\sqcap^{2\mathrm{k}} I)\in\breve{\mathbf{\Pi}}^{\circ}\,\square\,(b\sqcap^{2\mathrm{k}} I)=(c^{I}\sqcap^{\mathrm{k}} c^{I})}}{b\in\breve{\mathbf{\Pi}}^{\circ}, b=(c\sqcap^{\mathrm{k}} c)\Rightarrow(c^{I}\sqcap^{\mathrm{k}} c^{I})\in\breve{\mathbf{\Pi}}}}{b\in\breve{\mathbf{\Pi}}^{\circ}\,\square\, b=(c\sqcap^{\mathrm{k}} c)\Rightarrow(c^{I}\sqcap^{\mathrm{k}} c^{I})\in\breve{\mathbf{\Pi}}}}{(c\sqcap^{\mathrm{k}} c)\in\breve{\mathbf{\Pi}}\Rightarrow(c^{I}\sqcap^{\mathrm{k}} c^{I})\in\breve{\mathbf{\Pi}}}}{s\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow(s\sqcap^{\mathrm{k}} s)\in\breve{\mathbf{\Pi}}}\,.$$

QED

PROPOSITION 3.6. *Inferences according to the following schemata are* $\mathbf{L^iD^Z_\lambda}$*-derivable with an increase of the* $\mathbf{Z}$*-grade indicated on the right:*

(3.6i) $$\frac{\Rightarrow A}{s \in \breve{\mathbf{\Pi}}^\circ \Rightarrow [A/s]}\ {\scriptstyle +1}\,;$$

(3.6ii) $$\frac{\Rightarrow A}{\Rightarrow \Box A}\ {\scriptstyle +1}\,;$$

(3.6iii) $$\frac{A \Rightarrow B}{s \in \breve{\mathbf{\Pi}}^\circ, [A/s] \Rightarrow [B/s]}\ {\scriptstyle +1}\,;$$

(3.6iv) $$\frac{(\mathrm{k}+1)\,[A] \Rightarrow B}{s \in \breve{\mathbf{\Pi}}^\circ, [A/s \sqcap^{\mathrm{k}} s] \Rightarrow [B/s]}\ {\scriptstyle +1}\,;$$

(3.6v) $$\frac{\Rightarrow A \qquad B, \Pi \Rightarrow C}{A \supset B, \Pi \Rightarrow C}\ {\scriptstyle +1}\,;$$

(3.6vi) $$\frac{\Gamma \Rightarrow A \qquad B, \Pi \Rightarrow C}{A \supset B, \Box\Gamma, \Pi \Rightarrow C}\ {\scriptstyle +2}\,;$$

(3.6vii) $$\frac{A \Rightarrow B}{\Box A \Rightarrow \Box B}\ {\scriptstyle +2}\,;$$

(3.6viii) $$\frac{A, B \Rightarrow C}{\Box A, \Box B \Rightarrow \Box C}\ {\scriptstyle +2}\,;$$

(3.6ix) $$\frac{2\,[A] \Rightarrow B}{\Box A \Rightarrow \Box B}\ {\scriptstyle +2}\,;$$

(3.6x) $$\frac{\mathrm{k}\,[A] \Rightarrow B}{\Box A \Rightarrow \Box B}\ {\scriptstyle +2}\,;$$

(3.6xi) $$\frac{\Box(s \in \breve{\mathbf{\Pi}}^\circ), \Gamma \Rightarrow C}{s \in \breve{\mathbf{\Pi}}^\circ, \Gamma \Rightarrow C}\ {\scriptstyle +4}\,.$$

Proof. Re 3.6i. This is a straightforward application of 3.3i employing 2.7ii:

$$\dfrac{\dfrac{\Rightarrow A}{\Rightarrow [A/I]} \qquad \dfrac{[A/a] \Rightarrow [A/a] \qquad \Rightarrow A}{[A/a] \Rightarrow [A/a^I]}}{s \in \breve{\mathbf{\Pi}}^\circ \Rightarrow [A/s]}\,{}^{+1}\,.$$

Re 3.6ii. Cf. 134.21ii in [21], p. 1834.
Re 3.6iii. Cf. 134.10ii in [21], p. 1830.
Re 3.6iv. I only show the case of k = 1. Employ 2.7iv:

$$\dfrac{\dfrac{A, A \Rightarrow B}{\overline{[A/I \sqcap I] \Rightarrow [B/I]}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[A/c], [A/c] \Rightarrow [A/c \sqcap c] \qquad [B/c] \Rightarrow [B/c]}{[A/c \sqcap c] \rightarrow [B/c], [A/c], [A/c] \Rightarrow [B/c]} \qquad A, A \Rightarrow B}{\overline{[A/c \sqcap c] \rightarrow [B/c], [A/c], [A/c], A, A \Rightarrow [B/c] \mathbin{\square} B}}}{[A/c \sqcap c] \rightarrow [B/c], [A/c^I], [A/c^I] \Rightarrow [B/c^I]}}{[A/c \sqcap c] \rightarrow [B/c], [A/c^I \sqcap c^I] \Rightarrow [B/c^I]}}{s \in \breve{\mathbf{\Pi}}^\circ, [A/s \sqcap s] \Rightarrow [B/s]}\,{}^{+1}\,.$$

Re 3.6v. Straightforward consequence of the definition of ⊃ and 3.6i. Cf. 135.20iv in [21], p. 1847.
Re 3.6vi. Essentially as for 3.6v only with an additional inference according to schema 3.3iv. Left to the reader.
Re 3.6vii. Employ 3.3iv:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{a \in \breve{\mathbf{\Pi}}^\circ \Rightarrow a \in \breve{\mathbf{\Pi}}^\circ \qquad \dfrac{A \Rightarrow B}{[A/a], a \in \breve{\mathbf{\Pi}}^\circ \Rightarrow [B/a]}\,{}^{+1}}{a \in \breve{\mathbf{\Pi}}^\circ \rightarrow [A/a], a \in \breve{\mathbf{\Pi}}^\circ, a \in \breve{\mathbf{\Pi}}^\circ \Rightarrow [B/a]}}{\bigwedge x(x \in \breve{\mathbf{\Pi}}^\circ \rightarrow [A/x]), a \in \breve{\mathbf{\Pi}}^\circ, a \in \breve{\mathbf{\Pi}}^\circ \Rightarrow [B/a]}}{\bigwedge x(x \in \breve{\mathbf{\Pi}}^\circ \rightarrow [A/x]), a \in \breve{\mathbf{\Pi}}^\circ \Rightarrow [B/a]}\,{}^{+1}}{\bigwedge x(x \in \breve{\mathbf{\Pi}}^\circ \rightarrow [A/x]) \Rightarrow a \in \breve{\mathbf{\Pi}}^\circ \rightarrow [B/a]}}{\bigwedge x(x \in \breve{\mathbf{\Pi}}^\circ \rightarrow [A/x]) \Rightarrow \bigwedge x(x \in \breve{\mathbf{\Pi}}^\circ \rightarrow [B/x])}\,.$$

Re 3.6viii. Employ 3.3iv:

$$\dfrac{\dfrac{\dfrac{\dfrac{a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow a\in\breve{\mathbf{\Pi}}^{\circ} \qquad \dfrac{a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow a\in\breve{\mathbf{\Pi}}^{\circ} \qquad \dfrac{A, B \Rightarrow C}{[A/a], [B/a], a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [C/a]}\;{\scriptstyle +1}}{[A/a], a\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [B/a], a\in\breve{\mathbf{\Pi}}^{\circ}, a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [C/a]}}{a\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [A/a], a\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [B/a], a\in\breve{\mathbf{\Pi}}^{\circ}, a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [C/a]}\;{\scriptstyle +1}}{\dfrac{\dfrac{\bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [A/x]), \bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [B/x]), a\in\breve{\mathbf{\Pi}}^{\circ}, a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [C/a])}{\bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [A/x]), \bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [B/x]), a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [C/a])}\;{\scriptstyle +1}}{\bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [A/x]), \bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [B/x]) \Rightarrow a\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [C/a]}}}{\bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [A/x]), \bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [B/x]) \Rightarrow \bigwedge x(x\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [C/x])}\ .$$

Re 3.6ix. Employ 3.6iv and 3.3iii:[18] Let $\mathfrak{Q} :\equiv *_1\in\breve{\mathbf{\Pi}}^{\circ} \mathbin{\scriptstyle\square} (*_1 \sqcap *_1)\in\breve{\mathbf{\Pi}}$

$$\dfrac{\dfrac{\Rightarrow \mathfrak{Q}[I] \qquad \mathfrak{Q}[c] \Rightarrow \mathfrak{Q}[c^I] \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{b\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow b\in\breve{\mathbf{\Pi}}^{\circ} \qquad \dfrac{\dfrac{A, A \Rightarrow B}{[A/a \sqcap a], a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [B/a]}\;{\scriptstyle +1}}{b = a \sqcap a, [A/b], a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [B/a]}}{b\in\breve{\mathbf{\Pi}}^{\circ}, b = a \sqcap a, b\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [A/b], a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [B/a]}\;{\scriptstyle +1}}{b\in\breve{\mathbf{\Pi}}^{\circ}, b = a \sqcap a, \square A, a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [B/a]}}{b\in\breve{\mathbf{\Pi}}^{\circ} \mathbin{\scriptstyle\square} b = a \sqcap a, \square A, a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [B/a]}}{\dfrac{(a \sqcap a)\in\breve{\mathbf{\Pi}}, \square A, a\in\breve{\mathbf{\Pi}}^{\circ} \Rightarrow [B/a]}{a\in\breve{\mathbf{\Pi}}^{\circ} \mathbin{\scriptstyle\square} (a \sqcap a)\in\breve{\mathbf{\Pi}}, \square A \Rightarrow [B/a]}}}{\dfrac{a\in\breve{\mathbf{\Pi}}^{\circ}, \square A \Rightarrow [B/a]}{\square A \Rightarrow a\in\breve{\mathbf{\Pi}}^{\circ} \rightarrow [B/a]}}\;{\scriptstyle +1}}{\square A \Rightarrow \square B}\ .$$

Re 3.6x. Proof by induction on k. Essentially as for 3.6ix; left to the reader.

[18] I include this proof here because the one in [21], p. 1842, is flawed: some $\breve{\mathbf{\Pi}}$ should be $\breve{\mathbf{\Pi}}^{\circ}$.

Re 3.6xi. Employ 3.3ii:

$$\dfrac{\dfrac{\Rightarrow I\in\breve{\mathbf{\Pi}}^{\circ}}{\Rightarrow\Box(I\in\breve{\mathbf{\Pi}}^{\circ})}{+1}\qquad\dfrac{c\in\breve{\mathbf{\Pi}}^{\circ}\Rightarrow c^{I}\in\breve{\mathbf{\Pi}}^{\circ}}{\Box(c\in\breve{\mathbf{\Pi}}^{\circ})\Rightarrow\Box(c^{I}\in\breve{\mathbf{\Pi}}^{\circ})}{+2}\qquad\Box(s\in\breve{\mathbf{\Pi}}^{\circ}),\Gamma\Rightarrow C}{s\in\breve{\mathbf{\Pi}}^{\circ},\Gamma\Rightarrow C}{+1}\,.$$

QED

4. Successor and induction

DEFINITIONS 4.1. (1) $0 :\equiv \emptyset$.
(2) $s' :\equiv \{\!\!\{0, s\}\!\!\}$.

REMARKS 4.2. (1) Note that this successor notion is an 'exclusive' one, *i.e.*, one formulated in terms of identity.
(2) The definition of the successor of a term s along the line of $\{s,\{s\}\}$ doesn't lend itself to proving

$$s' = t' \to s = t$$

without induction. All that I was able to get is

$$s\in\mathbf{T}\sqcap t\in\mathbf{T}\sqcap s' = t' \to s = t\,.$$

This is why I adopt the above notion of the successor which allows the proof of 4.3vii without employing induction (and without employing any structural rules as shown in the next proposition).

PROPOSITION 4.3. *The following is* $\mathbf{L^{i}D}_{\lambda}$*-deducible:*

(4.3i) $s\in 0 \Rightarrow$;

(4.3ii) $\Rightarrow \{\!\!\{\{s\}\}\!\!\}\in s'$;

(4.3iii) $s' = 0 \Rightarrow$;

(4.3iv) $\{\!\!\{\{s\}\}\!\!\}\in t' \Rightarrow s\equiv t$;

(4.3v) $s' = t' \Rightarrow s \equiv t$;

(4.3vi) $s' = t' \Rightarrow s' \equiv t'$;

(4.3vii) $\mathfrak{F}[s], s' = t' \Rightarrow \mathfrak{F}[t]$.

Proof. Re 4.3i.

$$\dfrac{\bot\Rightarrow}{s\in\lambda\bot\Rightarrow}\,.$$

Re 4.3ii.

$$\dfrac{\dfrac{\Rightarrow \{\{s\}\} = \{\{s\}\}}{\Rightarrow \{\{s\}\} = \{\{0\},0\} \lor \{\{s\}\} = \{\{s\}\}}}{\Rightarrow \{\{s\}\} \in \{\{\{0\},0\},\{\{s\}\}\}}\,.$$

Re 4.3iii. Employ 4.3i:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow \{s\} \equiv \{\{s\}\}}{\Rightarrow \{\{s\}\} \equiv \{\{0\},0\} \lor \{\{s\}\} \equiv \{\{s\}\}}}{\Rightarrow \{\{s\}\} \in \{\{\{0\},0\},\{\{s\}\}\}} \qquad \{\{s\}\} \in 0 \Rightarrow}{\{\{s\}\} \in s' \rightarrow \{\{s\}\} \in 0 \Rightarrow}}{\{\{s\}\} \in s' \leftrightarrow \{\{s\}\} \in 0 \Rightarrow}}{\bigwedge x\,(x \in s' \leftrightarrow x \in 0) \Rightarrow}\,.$$

Re 4.3iv. Cf. [21], 127.35iv, p. 1745.
Re 4.3v. Employ 4.3ii and 4.3iv:

$$\dfrac{\dfrac{\dfrac{\Rightarrow \{\{s\}\} \in s' \qquad \{\{s\}\} \in t' \Rightarrow s \equiv t}{\{\{s\}\} \in s' \rightarrow \{\{s\}\} \in t' \Rightarrow s \equiv t}}{\{\{s\}\} \in s' \leftrightarrow \{\{s\}\} \in t' \Rightarrow s \equiv t}}{s' = t' \Rightarrow s \equiv t}\,.$$

Re 4.3vi and 4.3vii. These are immediate consequences of 4.3v. QED

DEFINITION 4.4. $\mathbf{N}^{\circ} :\equiv \lambda x \bigwedge y (\bigwedge z (z \in y \rightarrow z' \in y) \supset (0 \in y \rightarrow x \in y))$.[19]

REMARKS 4.5. (1) $\mathbf{N}^{\circ}$ is what I called an *exclusive* notion, *e.g.*, in [21], p. 1596, remark 116.6: it only contains the *numerals* $0, 0', 0'', \ldots$ and nothing else that may have the same numerical value but isn't really the same term, like, for instance, $0+0$.[20] This not only provides for the contractibility of wffs of the form $s \in \mathbf{N}^{\circ}$, but also for the possibility of proving a form of induction.

[19] Note the difference of the foregoing definition to that in [20], p. 400 (positioning of "step" and "basis": this is to get the "basis" from the left to the right side of weak implication).

[20] This is the difference to Frege and also Quine. Needless to say, that for them it is a confusion to make such a distinction and, thereby, to object to their conflation.

(2) As an immediate consequence of 4.3v, one has

$$s \in \mathbf{N}^{\circ}, s' = t' \Rightarrow t \in \mathbf{N}^{\circ} .$$

PROPOSITION 4.6. *The following is* $\mathbf{L^iD}_{\lambda}$*-deducible:*

(4.6i) $\Rightarrow 0 \in \mathbf{N}^{\circ}$;

(4.6ii) $s \in \mathbf{N}^{\circ} \Rightarrow s' \in \mathbf{N}^{\circ}$.

Proof. Pretty trivial and for that reason left to the reader; but the point to note is that 4.6ii does not require any **Z**-inference despite involving weak implication. This is a straightforward consequence of 2.16. QED

As in the case of proto-induction, I provide a list of schemata for derivable inferences together with an indication of how many **Z**-inferences go into it.

PROPOSITION 4.7. *Inferences according to the following schemata are* $\mathbf{L^iD}^{\mathrm{Z}}_{\lambda}$*-derivable with an increase of the* **Z***-grade indicated on the right:*

(4.7i) $$\frac{\Gamma \Rightarrow \mathfrak{F}[0] \qquad \mathfrak{F}[a] \Rightarrow \mathfrak{F}[a']}{s \in \mathbf{N}^{\circ}, \Gamma \Rightarrow \mathfrak{F}[s]} \,{+1}\,;$$

(4.7ii) $$\frac{\Gamma \Rightarrow \mathfrak{F}[0] \qquad \mathfrak{F}[a] \Rightarrow \mathfrak{F}[a'] \qquad \mathfrak{F}[s], \Pi \Rightarrow C}{s \in \mathbf{N}^{\circ}, \Gamma, \Pi \Rightarrow C} \,{+1}\,;$$

(4.7iii) $$\frac{\Gamma \Rightarrow \mathfrak{F}[a, 0] \qquad \mathfrak{F}[s, b] \Rightarrow \mathfrak{F}[a, b']}{t \in \mathbf{N}^{\circ} \Rightarrow \mathfrak{F}[s, t]} \,{+1}\,;$$

(4.7iv) $$\frac{s \in \mathbf{N}^{\circ}, s \in \mathbf{N}^{\circ}, \Gamma \Rightarrow C}{s \in \mathbf{N}^{\circ}, \Gamma \Rightarrow C} \,{+1}\,;$$

(4.7v) $$\frac{\Gamma \Rightarrow \mathfrak{F}[0] \qquad a \in \mathbf{N}^{\circ}, \mathfrak{F}[a] \Rightarrow \mathfrak{F}[a']}{s \in \mathbf{N}^{\circ}, \Gamma \Rightarrow \mathfrak{F}[s]} \,{+2}\,;$$

(4.7vi) $$\frac{\Box(s \in \mathbf{N}^{\circ}), \Gamma \Rightarrow C}{s \in \mathbf{N}^{\circ}, \Gamma \Rightarrow C} \,{+4}\,;$$

(4.7vii) $$\frac{\Gamma \Rightarrow \mathfrak{F}[0] \qquad \mathrm{k}[\mathfrak{F}[a]] \Rightarrow \mathfrak{F}[a']}{s \in \mathbf{N}^{\circ} \Rightarrow \mathfrak{F}[s]} \,{+4}\,;$$

(4.7viii)
$$\dfrac{s\in\mathbf{N}^{\circ} \Rightarrow \mathfrak{F}[s,0] \qquad \mathrm{k}[\mathfrak{F}[s,a]] \Rightarrow \mathfrak{F}[s,a'] \qquad \mathfrak{F}[s,t], \mathit{\Gamma} \Rightarrow C}{s\in\mathbf{N}^{\circ}, t\in\mathbf{N}^{\circ}, \mathit{\Gamma} \Rightarrow C}\,{\scriptstyle +9}\,;$$

(4.7ix)
$$\dfrac{\mathit{\Gamma} \Rightarrow \mathfrak{F}[0,b] \qquad \Rightarrow \mathfrak{F}[a',0] \qquad \mathfrak{F}[a,b] \Rightarrow \mathfrak{F}[a',b']}{s\in\mathbf{N}, t\in\mathbf{B} \Rightarrow \mathfrak{F}[s,t]}\,{\scriptstyle +2}\,.$$

Proof. Re 4.7i. This is a straightforward consequence of the way $\mathbf{N}^{\circ}$ is defined, employing 3.6v:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathfrak{F}[a] \Rightarrow \mathfrak{F}[a']}{a\in\lambda x\,\mathfrak{F}[x] \Rightarrow a'\in\lambda x\,\mathfrak{F}[x]}}{\Rightarrow a\in\lambda x\,\mathfrak{F}[x] \to a'\in\lambda x\,\mathfrak{F}[x]}}{\Rightarrow \bigwedge z(z\in\lambda x\,\mathfrak{F}[x] \to z'\in\lambda x\,\mathfrak{F}[x])} \qquad \dfrac{\dfrac{\mathit{\Gamma} \Rightarrow \mathfrak{F}[0]}{\mathit{\Gamma} \Rightarrow 0\in\lambda x\,\mathfrak{F}[x]} \qquad \dfrac{\mathfrak{F}[s] \Rightarrow \mathfrak{F}[s]}{s\in\lambda x\,\mathfrak{F}[x] \Rightarrow \mathfrak{F}[s]}}{0\in\lambda x\,\mathfrak{F}[x] \to s\in\lambda x\,\mathfrak{F}[x], \mathit{\Gamma} \Rightarrow \mathfrak{F}[s]}}{\bigwedge z(z\in\lambda x\,\mathfrak{F}[x] \to z'\in\lambda x\,\mathfrak{F}[x]) \supset (0\in\lambda x\,\mathfrak{F}[x] \to s\in\lambda x\,\mathfrak{F}[x]), \mathit{\Gamma} \Rightarrow \mathfrak{F}[s]}{\scriptstyle +1}}{\bigwedge y(\bigwedge z(z\in y \to z'\in y) \supset (0\in y \to s\in y)), \mathit{\Gamma} \Rightarrow \mathfrak{F}[s]}}{s\in\lambda x \bigwedge y(\bigwedge z(z\in y \to z'\in y) \supset (0\in y \to x\in y)), \mathit{\Gamma} \Rightarrow \mathfrak{F}[s]}\,.$$

Re 4.7ii. This is also a straightforward consequence of the way $\mathbf{N}^{\circ}$ is defined, employing 3.6v:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\genfrac{}{}{1.2pt}{}{\mathfrak{F}[a] \Rightarrow \mathfrak{F}[a']}{a\in\lambda x\,\mathfrak{F}[x] \Rightarrow a'\in\lambda x\,\mathfrak{F}[x]}}{\Rightarrow a\in\lambda x\,\mathfrak{F}[x] \to a'\in\lambda x\,\mathfrak{F}[x]}}{\Rightarrow \bigwedge z(z\in\lambda x\,\mathfrak{F}[x] \to z'\in\lambda x\,\mathfrak{F}[x])} \qquad \dfrac{\dfrac{\mathit{\Gamma} \Rightarrow \mathfrak{F}[0]}{\mathit{\Gamma} \Rightarrow 0\in\lambda x\,\mathfrak{F}[x]} \qquad \dfrac{\mathfrak{F}[s], \mathit{\Pi} \Rightarrow C}{s\in\lambda x\,\mathfrak{F}[x], \mathit{\Pi} \Rightarrow C}}{0\in\lambda x\,\mathfrak{F}[x] \to s\in\lambda x\,\mathfrak{F}[x], \mathit{\Gamma}, \mathit{\Pi} \Rightarrow C}}{\bigwedge z(z\in\lambda x\,\mathfrak{F}[x] \to z'\in\lambda x\,\mathfrak{F}[x]) \supset (0\in\lambda x\,\mathfrak{F}[x] \to s\in\lambda x\,\mathfrak{F}[x]), \mathit{\Gamma}, \mathit{\Pi} \Rightarrow C}{\scriptstyle +1}}{\bigwedge y(\bigwedge z(z\in y \to z'\in y) \supset (0\in y \to s\in y)), \mathit{\Gamma}, \mathit{\Pi} \Rightarrow C}}{s\in\lambda x \bigwedge y(\bigwedge z(z\in y \to z'\in y) \supset (0\in y \to x\in y)), \mathit{\Gamma}, \mathit{\Pi} \Rightarrow C}\,.$$

Re 4.7iii. Employ 4.7ii:

$$\dfrac{\dfrac{\mathit{\Gamma} \Rightarrow \mathfrak{F}[a,0]}{\mathit{\Gamma} \Rightarrow \bigwedge x\,\mathfrak{F}[x,0]} \qquad \dfrac{\dfrac{\mathfrak{F}[s,b] \Rightarrow \mathfrak{F}[a,b']}{\bigwedge x\,\mathfrak{F}[x,b] \Rightarrow \mathfrak{F}[a,b']}}{\bigwedge x\,\mathfrak{F}[x,b] \Rightarrow \bigwedge x\,\mathfrak{F}[x,b']} \qquad \dfrac{\mathfrak{F}[s,t] \Rightarrow \mathfrak{F}[s,t]}{\bigwedge x\,\mathfrak{F}[x,t] \Rightarrow \mathfrak{F}[s,t]}}{t\in\mathbf{N}^{\circ}, \mathit{\Gamma} \Rightarrow \mathfrak{F}[s,t]}\,{\scriptstyle +1}\,.$$

Re 4.7iv. As for 3.3iv only with 4.7ii instead of 3.3iii.
Re 4.7v. As for 3.3v only with 4.7iv and 4.7ii instead of 3.3v and 3.3iii.
Re 4.7vi. As for 3.6xi only with 4.7iv and 4.7ii instead of 3.3v and 3.3iii.
Re 4.7vii. Employ 3.6x, 4.7iv and 3.6v with an inference according to 4.7ii:

$$\dfrac{\dfrac{\Rightarrow \mathfrak{F}[0]}{\Rightarrow \Box\,\mathfrak{F}[0]}{\scriptstyle +1} \qquad \dfrac{\mathrm{k}[\mathfrak{F}[b]] \Rightarrow \mathfrak{F}[b']}{\Box\,\mathfrak{F}[b] \Rightarrow \Box\,\mathfrak{F}[b']}{\scriptstyle +2} \qquad \dfrac{\mathfrak{F}[t] \Rightarrow \mathfrak{F}[t]}{\Box\,\mathfrak{F}[t] \Rightarrow \mathfrak{F}[t]}}{t \in \mathbf{N}^{\circ} \Rightarrow \mathfrak{F}[t]}{\scriptstyle +1}\,.$$

Re 4.7viii. Employ 3.6vii, 4.7vi, 3.6x and 4.7ii:

$$\dfrac{\dfrac{\dfrac{s \in \mathbf{N}^{\circ} \Rightarrow \mathfrak{F}[s,0]}{\Box(s \in \mathbf{N}^{\circ}) \Rightarrow \Box\,\mathfrak{F}[s,0]}{\scriptstyle +2}}{s \in \mathbf{N}^{\circ} \Rightarrow \Box\,\mathfrak{F}[s,0]}{\scriptstyle +4} \qquad \dfrac{\mathrm{k}[\mathfrak{F}[s,a]] \Rightarrow \mathfrak{F}[s,a']}{\Box\,\mathfrak{F}[s,a] \Rightarrow \Box\,\mathfrak{F}[s,a']}{\scriptstyle +2} \qquad \dfrac{\mathfrak{F}[s,t], \Gamma \Rightarrow C}{\Box\,\mathfrak{F}[s,t], \Gamma \Rightarrow C}}{s \in \mathbf{N}^{\circ}, t \in \mathbf{N}^{\circ}, \Gamma \Rightarrow C}{\scriptstyle +1}\,.$$

Re 4.7ix. Let β be a fixed point: $\beta = \lambda x(x \equiv 0 \vee \bigvee y(y \in \beta \mathbin{\scriptstyle\Box} y' \equiv x))$. First, the following deduction is in $\mathbf{L^{i}D}_{\lambda}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow \mathfrak{F}[a',0]}{c \equiv 0 \Rightarrow \mathfrak{F}[a',c]} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\mathfrak{F}[a,b] \Rightarrow \mathfrak{F}[a',b']}{b \in \beta, \bigwedge y(y \in \beta \to \mathfrak{F}[a,y]) \Rightarrow \mathfrak{F}[a',b']}}{b \in \beta, b' \equiv c, \bigwedge y(y \in \beta \to \mathfrak{F}[a,y]) \Rightarrow \mathfrak{F}[a',c]}}{b \in \beta \mathbin{\scriptstyle\Box} b' \equiv c, \bigwedge y(y \in \beta \to \mathfrak{F}[a,y]) \Rightarrow \mathfrak{F}[a',c]}}{\bigvee y(y \in \beta \mathbin{\scriptstyle\Box} y' \equiv c), \bigwedge y(y \in \beta \to \mathfrak{F}[a,y]) \Rightarrow \mathfrak{F}[a',c]}}{c \equiv 0 \vee \bigvee y(y \in \beta \mathbin{\scriptstyle\Box} y' \equiv c), \bigwedge y(y \in \beta \to \mathfrak{F}[a,y]) \Rightarrow \mathfrak{F}[a',c]}}{c \in \beta, \bigwedge y(y \in \beta \to \mathfrak{F}[a,y]) \Rightarrow \mathfrak{F}[a',c]}}{\bigwedge y(y \in \beta \to \mathfrak{F}[a,y]) \Rightarrow c \in \beta \to \mathfrak{F}[a',c]}}{\bigwedge y(y \in \beta \to \mathfrak{F}[a,y]) \Rightarrow \bigwedge y(y \in \beta \to \mathfrak{F}[a',y])}\,.$$

The next step is to establish the following, employing one **Z**-inference:

$$\dfrac{\dfrac{\dfrac{\Rightarrow 0\in\beta \qquad \dfrac{\dfrac{\dfrac{\dfrac{a\in\beta\Rightarrow a\in\beta \qquad \Rightarrow a'\equiv a'}{a\in\beta\Rightarrow a\in\beta\mathbin{\Box} a'\equiv a'}}{a\in\beta \Rightarrow \bigvee y\,(y\in\beta\mathbin{\Box}y'\equiv a')}}{a\in\beta\Rightarrow a'\equiv 0\vee\bigvee y\,(y\in\beta\mathbin{\Box} y'\equiv a')}}{a\in\beta\Rightarrow a'\in\beta}}{t\in\mathbf{N}^\circ\Rightarrow t\in\beta}{\scriptstyle +1} \qquad \mathfrak{F}[s,t]\Rightarrow\mathfrak{F}[s,t]}{t\in\beta\to\mathfrak{F}[s,t],t\in\mathbf{N}^\circ\Rightarrow\mathfrak{F}[s,t]}}{\bigwedge y\,(y\in\beta\to\mathfrak{F}[s,y]),t\in\mathbf{N}^\circ\Rightarrow\mathfrak{F}[s,t]\mathfrak{F}[s,y])}\ .$$

Finally an inference according to 4.7ii. Let $\bigwedge^\beta y\,\mathfrak{F}[y] :\equiv \bigwedge x\,(y\in\beta\to\mathfrak{F}[y])$ to save space:

$$\dfrac{\dfrac{\varGamma\Rightarrow\mathfrak{F}[0,b]}{\varGamma\Rightarrow\bigwedge^\beta y\,\mathfrak{F}[0,y]} \quad \bigwedge^\beta y\,\mathfrak{F}[a,y]\Rightarrow\bigwedge^\beta y\,\mathfrak{F}[a',y] \quad \bigwedge^\beta y\,\mathfrak{F}[s,y],t\in\mathbf{N}^\circ\Rightarrow\mathfrak{F}[s,t]}{s\in\mathbf{N},t\in\mathbf{N}^\circ,\varGamma\Rightarrow\mathfrak{F}[s,t]}{\scriptstyle +1}\ .$$

QED

REMARKS 4.8. (1) 4.7i is a straightforward consequence of the way $\mathbf{N}^\circ$ is defined and is just the usual second order way of dealing with induction which is actually all that is needed in the classical case. Without contraction, however, this is not yet quite sufficient for proving the relevant properties of primitive recursive functions and this is why the further schemata are introduced.

(2) 4.7ii is designed to avoid cuts that would become necessary if 4.7i were employed in the case, *e.g.*, of the totality proofs below.

(3) 4.7iii is discussed in [13], p. 348, under the label *Erweiterung des Induktionsschemas* ("extension of the schema of induction") and is only listed here for the sake of interest.

(4) 4.7v deals with the situation that the induction step in turn depends on the free variable having only values in $\mathbf{N}^\circ$.

(5) 4.7vii deals with the situation that the "induction step" requires the induction hypothesis more than once. Suppose, for instance, that all we can get is

$$\Rightarrow\mathfrak{F}[0] \qquad \text{and} \qquad 2[\mathfrak{F}[a]\Rightarrow\mathfrak{F}[a']\,.$$

Then, in order to get to $\Rightarrow\mathfrak{F}[3]$, we need the induction basis 2^3-times:

$$8[\mathfrak{F}[0]] \Rightarrow 4[\mathfrak{F}[1]]\,, \quad 4[\mathfrak{F}[1]] \Rightarrow 2[\mathfrak{F}[2]]\,, \quad 2[\mathfrak{F}[2]] \Rightarrow \mathfrak{F}[3]\,.$$

That's what is here being accounted for by the necessity notion:

$$\frac{\mathrm{k}[\mathfrak{F}[a]] \Rightarrow \mathfrak{F}[a']}{\Box\,\mathfrak{F}[a] \Rightarrow \Box\,\mathfrak{F}[a']}\;.$$

One might think of defining

$$\mathbf{N}^{2^\circ} :\equiv \lambda x \bigwedge y (\bigwedge z ([z\in y]^{\cdot 2} \rightarrow z'\in y) \supset (0\in y \rightarrow x\in y))$$

in order to have an induction that tolerates two assumptions in the induction step. This would give

$$\frac{\Gamma \Rightarrow \mathfrak{F}[0] \qquad \mathrm{k}[\mathfrak{F}[a]] \Rightarrow \mathfrak{F}[a']}{s\in[\mathbf{N}^\circ]^{\mathrm{k}} \Rightarrow \mathfrak{F}[s]}\;,$$

but not $s\in\mathbf{N}^{2^\circ} \Rightarrow s'\in\mathbf{N}^{2^\circ}$, which makes the whole thing useless.
(6) As 4.7ii, 4.7viii is designed to avoid cuts that would become necessary if 4.7i were employed.
(7) 4.7ix is a "double induction" without "nesting". My reason to include it here is that, unlike "nested double induction", it is perfectly $\mathbf{L}^{\mathrm{i}}\mathbf{D}^{\mathrm{Z}}_{\lambda}$-derivable. The following "nested schema of induction" (without side wffs) is discussed in [13], p. 352 (*verschränktes Induktionsschema*):

$$\frac{\Rightarrow \mathfrak{F}[0,b] \qquad \mathfrak{F}[a,t_1] \Rightarrow \mathfrak{F}[a',0] \qquad \mathfrak{F}[a,t_2],\mathfrak{F}[a',b] \Rightarrow \mathfrak{F}[a',b']}{s\in\mathbf{N}, t\in\mathbf{N}, \Gamma, \Pi, \Xi \Rightarrow \mathfrak{F}[s,t]}\;.$$

This does not only require two inductions, but also a side wff in the first induction which, in the dialectical case, can only be accommodated for by introducing a necessity operator $\Box$ which spoils the classical reduction:

$$\dfrac{\dfrac{\Rightarrow \mathfrak{F}[0,b]}{\Rightarrow \bigwedge y\,\mathfrak{F}[0,b]} \qquad \dfrac{\dfrac{\dfrac{\mathfrak{F}[a,t_1] \Rightarrow \mathfrak{F}[a',0]}{\bigwedge y\,\mathfrak{F}[a,y] \Rightarrow \mathfrak{F}[a',0]} \qquad \dfrac{\mathfrak{F}[a,t_2],\mathfrak{F}[a',b] \Rightarrow \mathfrak{F}[a',b']}{\bigwedge y\,\mathfrak{F}[a,y],\mathfrak{F}[a',b] \Rightarrow \mathfrak{F}[a',b']}}{\bigwedge y\,\mathfrak{F}[a,y] \Rightarrow \mathfrak{F}[a',c]}}{\bigwedge y\,\mathfrak{F}[a,y] \Rightarrow \bigwedge y\,\mathfrak{F}[a',y]}}{\dfrac{\Rightarrow \bigwedge y\,\mathfrak{F}[s,y]}{\Rightarrow \mathfrak{F}[s,t]}}\;.$$

How this can be treated will be the topic of another paper, following the approach of my [23], section 5, pp. 136–159.

Proposition 4.9. $\mathbf{L^iD}_\lambda^{Z\restriction_1} \vdash\ s' \in \mathbf{N}^\circ \Rightarrow s \in \mathbf{N}^\circ$.

Proof. Re 4.9. This is the reversal of 4.6ii; it requires a **Z**-inference. Let $\mathfrak{N} :\equiv 0 \equiv *_1 \vee \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv *_1)$ and show $\Rightarrow \mathfrak{N}[0]$, $\mathfrak{N}[c] \Rightarrow \mathfrak{N}[c']$, and $\mathfrak{N}[s'] \Rightarrow s \in \mathbf{N}^\circ$:

$$\frac{\Rightarrow 0 \equiv 0}{\Rightarrow 0 \equiv 0 \vee \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv 0)}\ .$$

$$\dfrac{\dfrac{\dfrac{\Rightarrow 0' \in \mathbf{N}^\circ \qquad c \equiv 0 \Rightarrow 0' \equiv c'}{c \equiv 0 \Rightarrow 0' \in \mathbf{N}^\circ \sqcap 0' \equiv c'}}{c \equiv 0 \Rightarrow \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c')} \qquad \dfrac{\dfrac{\dfrac{b \in \mathbf{N}^\circ, b' \equiv c \Rightarrow b' \in \mathbf{N}^\circ \sqcap b'' \equiv c'}{b \in \mathbf{N}^\circ, b' \equiv c \Rightarrow \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c')}}{b \in \mathbf{N}^\circ \sqcap b' \equiv c \Rightarrow \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c')}}{\bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c) \Rightarrow \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c')}}{\dfrac{c \equiv 0 \vee \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c) \Rightarrow \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c')}{c \equiv 0 \vee \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c) \Rightarrow c' \equiv 0 \vee (\bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv c')}}\ .$$

Now employ 4.3iii. and 4.3v:

$$\dfrac{s' \equiv 0 \Rightarrow \qquad \dfrac{\dfrac{b \in \mathbf{N}^\circ, b' \equiv s' \Rightarrow s \in \mathbf{N}^\circ}{b \in \mathbf{N}^\circ \sqcap b' \equiv s' \Rightarrow s \in \mathbf{N}^\circ}}{\bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv s') \Rightarrow s \in \mathbf{N}^\circ}}{s' \equiv 0 \vee \bigvee y(y \in \mathbf{N}^\circ \sqcap y' \equiv s') \Rightarrow s \in \mathbf{N}^\circ}\ .$$

QED

Remark 4.10. The successor operation from definition 4.1 (2) can be turned into a successor *function* more in tune with the other definitions of functions that are to come:

$$\mathbf{S} :\equiv \lambda x_1 x_2 (x_2 = x_1')\ .$$

Obviously, $\mathbf{S}[\![t]\!] = t'$ would then be $\mathbf{L^iD}_\lambda$-deducible. Due to the exclusive character of $\mathbf{N}^\circ$, however, $s \in \mathbf{N}^\circ \rightarrow \mathbf{S}[\![s]\!] \in \mathbf{N}^\circ$ would not hold, only $s \in \mathbf{N} \rightarrow \mathbf{S}[\![s]\!] \in \mathbf{N}$, where $\mathbf{N}$ is defined as in 5.3 below.

5. Predecessor

The predecessor function can be defined without employing the fixed point property.

Definition 5.1. $pd :\equiv \lambda xy((x = 0 \wedge y = 0) \vee \bigvee z(z' = x \sqcap z = y))$.

PROPOSITION 5.2. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(5.2i) $\Rightarrow pd\llbracket 0\rrbracket = 0\,;$

(5.2ii) $a\in s, \langle s', b\rangle \in pd \Rightarrow a\in b\,;$

(5.2iii) $\Rightarrow \langle s', s\rangle\,;$

(5.2iv) $\Rightarrow pd\llbracket s'\rrbracket = s\,.$

Proof. Re 5.2i

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow 0=0 \qquad \Rightarrow 0=0}{\Rightarrow 0=0\wedge 0=0}}{\Rightarrow (0=0\wedge 0=0)\vee \bigvee z\,(z'=0\,\square\, z=0)}}{\Rightarrow \langle 0,0\rangle\in pd} \qquad a\in 0\Rightarrow a\in s}{\langle 0,0\rangle\in pd \to a\in 0\Rightarrow a\in s}}{\bigwedge y\,(\langle 0,y\rangle\in pd\to a\in y)\Rightarrow a\in s}}{a\in pd\llbracket 0\rrbracket\Rightarrow a\in 0} \qquad a\in 0\Rightarrow a\in pd\llbracket 0\rrbracket}{\Rightarrow pd\llbracket 0\rrbracket = 0}\,.$$

Re 5.2ii. Employ 4.3ii and 4.3i:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow \{\{s\}\}\in s' \qquad \{\{s\}\}\in 0\Rightarrow}{\{\{s\}\}\in s'\to \{\{s\}\}\in 0\Rightarrow}}{s'=0\Rightarrow}}{s'=0\wedge b=0\Rightarrow} \qquad \dfrac{\dfrac{\dfrac{\dfrac{a\in s, s=b\Rightarrow a\in b}{a\in s, c=s, c=b\Rightarrow a\in b}}{a\in s, c'=s', c=b\Rightarrow a\in b}}{a\in s, c'=s'\,\square\, c=b\Rightarrow a\in b}}{a\in s, \bigvee z\,(z'=s'\,\square\, z=b)\Rightarrow a\in b}}{a\in s, (s'=0\wedge b=0)\vee\bigvee z\,(z'=s'\,\square\, z=b)\Rightarrow a\in b}}{a\in s, \langle s', b\rangle\in pd\Rightarrow a\in b}\,.$$

Re 5.2iii.

$$\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow s'=s' \qquad \Rightarrow s=s}{\Rightarrow s'=s'\,\square\, s=s}}{\Rightarrow \bigvee z\,(z'=s'\,\square\, z=s)}}{\Rightarrow (s'=0\wedge s=0)\vee \bigvee z\,(z'=s'\,\square\, z=s)}}{\Rightarrow \langle s', s\rangle\in pd}\,.$$

Re 5.2iv. Employ 5.2iii and 5.2ii:

$$\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow \langle s', s\rangle \in pd \qquad a\in s \Rightarrow a\in s}{\langle s', s\rangle \in pd \to a\in s \Rightarrow a\in s}}{\bigwedge y\,(\langle s', y\rangle \in pd \to a\in y) \Rightarrow a\in s}}{a\in pd[\![s']\!] \Rightarrow a\in s} \qquad \dfrac{\dfrac{\dfrac{a\in s, \langle s', b\rangle \in pd \Rightarrow a\in b}{a\in s \Rightarrow \langle s', b\rangle \in pd \to a\in b}}{a\in s \Rightarrow \bigwedge y\,(\langle s', y\rangle \in pd \to a\in y)}}{a\in s \Rightarrow a\in pd[\![s']\!]}}{\Rightarrow pd[\![s']\!] = s} \;.$$

QED

Next comes the totality of the predecessor function. It will be obvious that totality can't hold for the predecessor function in the sense that it does for the successor operation as established in 4.6i: $pd[\![s]\!]$ just isn't in $\mathbf{N}^{\circ}$, no matter what its numerical value.[21] So in order to be able to establish some sort of totality we will have to shift to an inclusive notion of natural numbers.

Definition 5.3. $\mathbf{N} :\equiv \lambda x \bigvee y\,(y\in\mathbf{N}^{\circ} \,\square\, x = y)$.[22]

Proposition 5.4. *The following is* $\mathbf{L}^{\mathbf{i}}\mathbf{D}_{\lambda}$*-deducible:*

(5.4i) $\Rightarrow 0\in\mathbf{N}^{\circ} \wedge pd[\![0]\!]\in\mathbf{N}\,;$

(5.4ii) $c\in\mathbf{N}^{\circ} \wedge pd[\![c]\!]\in\mathbf{N} \Rightarrow c'\in\mathbf{N}^{\circ} \wedge pd[\![c']\!]\in\mathbf{N}\,;$

(5.4iii) $pd[\![a]\!]\in\mathbf{N} \wedge a\in\mathbf{N}^{\circ}, s = a \Rightarrow pd[\![s]\!]\in\mathbf{N}\,.$

Proof. Re 5.4i. Employ 5.2i:

$$\dfrac{\Rightarrow 0\in\mathbf{N}^{\circ} \qquad \dfrac{\dfrac{\dfrac{\Rightarrow 0\in\mathbf{N}^{\circ} \qquad \Rightarrow pd[\![0]\!] = 0}{\Rightarrow 0\in\mathbf{N}^{\circ} \,\square\, pd[\![0]\!] = 0}}{\Rightarrow \bigvee y\,(y\in\mathbf{N}^{\circ} \,\square\, pd[\![0]\!] = y)}}{\Rightarrow pd[\![0]\!]\in\mathbf{N}}}{\Rightarrow 0\in\mathbf{N}^{\circ} \wedge pd[\![0]\!]\in\mathbf{N}} \;.$$

[21] Cf. remark 4.5 (1) above.

[22] In [21], p. 1881, definition 136.48, I introduced a notion of $\mathbf{N}$ that involved $\square$ and in that way provided for more than just one substitution. This, however, is not needed in the present context and since it is likely to be the source of some increase of $\mathbf{Z}$-inferences we may well stick to a more restricted notion — as actually in the case of $\breve{\mathbf{\Pi}}$.

Re 5.4ii. Employ 5.2iv:

$$\dfrac{c\in\mathbf{N}^\circ \Rightarrow c'\in\mathbf{N}^\circ \qquad \dfrac{\dfrac{\dfrac{c\in\mathbf{N}^\circ \Rightarrow c\in\mathbf{N}^\circ \quad \Rightarrow pd\llbracket c'\rrbracket = c}{c\in\mathbf{N}^\circ \Rightarrow c\in\mathbf{N}^\circ \,\square\, pd\llbracket c'\rrbracket = c}}{c\in\mathbf{N}^\circ \Rightarrow \bigvee y\,(y\in\mathbf{N}^\circ \,\square\, pd\llbracket c'\rrbracket = y)}}{c\in\mathbf{N}^\circ \Rightarrow pd\llbracket c'\rrbracket\in\mathbf{N}}}{\dfrac{c\in\mathbf{N}^\circ \Rightarrow c'\in\mathbf{N}^\circ \wedge pd\llbracket c'\rrbracket\in\mathbf{N}}{c\in\mathbf{N}^\circ \wedge pd\llbracket c\rrbracket\in\mathbf{N} \Rightarrow c'\in\mathbf{N}^\circ \wedge pd\llbracket c'\rrbracket\in\mathbf{N}}}\ .$$

Re 5.4iii.

$$\begin{array}{c}
b\in\mathbf{N}^\circ, pd\llbracket s\rrbracket = b \Rightarrow b\in\mathbf{N}^\circ \,\square\, pd\llbracket s\rrbracket = b \\ \hline
b\in\mathbf{N}^\circ, pd\llbracket s\rrbracket = b \Rightarrow \bigvee y\,(y\in\mathbf{N}^\circ \,\square\, pd\llbracket s\rrbracket = y) \\ \hline
b\in\mathbf{N}^\circ, pd\llbracket s\rrbracket = b \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N} \\ \hline
b\in\mathbf{N}^\circ, pd\llbracket a\rrbracket = b, s = a \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N} \\ \hline
b\in\mathbf{N}^\circ \,\square\, pd\llbracket a\rrbracket = b, s = a \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N} \\ \hline
\bigvee y\,(y\in\mathbf{N}^\circ \,\square\, pd\llbracket a\rrbracket = y), s = a \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N} \\ \hline
pd\llbracket a\rrbracket\in\mathbf{N}, s = a \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N} \\ \hline
a\in\mathbf{N}^\circ \wedge pd\llbracket a\rrbracket\in\mathbf{N}, s = a \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N}
\end{array}\ .$$

QED

PROPOSITION 5.5. $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction_1} \vdash\ s\in\mathbf{N} \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N}$.

Proof. Employ 5.4i–5.4iii with an induction inference according to 4.7ii and continue as follows:

$$\begin{array}{c}
a\in\mathbf{N}^\circ, s = a \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N} \\ \hline
a\in\mathbf{N}^\circ \,\square\, s = a \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N} \\ \hline
\bigvee y\,(y\in\mathbf{N}^\circ \,\square\, s = y) \Rightarrow pd\llbracket s\rrbracket\in\mathbf{N}
\end{array}\ .$$

QED

6. Recursion equations for addition

PROPOSITION 6.1. *There exists a term $\mathcal{A}$ such that:*

$$\mathbf{L^iD}_\lambda \vdash \mathcal{A} = \lambda x_1x_2x_3\,((x_2 = 0 \wedge x_3 = x_1) \vee \\ \bigvee y_1 \bigvee y_2 \bigvee y_3\,(y_1 = x_1 \,\square\, y_2' = x_2 \,\square\, y_3' = x_3 \,\square\, \langle\!\langle\langle\!\langle y_1, y_2\rangle\!\rangle, y_3\rangle\!\rangle\in\mathcal{A})\,.$$

Proof. This is an immediate consequence of the fixed-point property. QED

CONVENTIONS 6.2. (1) The following abbreviation is introduced to simplify presentation:

$$\mathcal{A}_s :\equiv \lambda x_2 x_3 \, (x_2 = 0 \wedge x_3 = s) \vee \\ \qquad\qquad \bigvee y_1 \bigvee y_2 \, (y_1' = x_2 \square y_2' = x_3 \square \langle\langle s, y_2\rangle, y_3\rangle \in \mathcal{A}) \, .$$

The full definition is only really needed in the proof of 6.7ii below.
(2) In order to save space in presentations, I shall occasionally use the following abbreviations:

$bas_{\mathcal{A}_s}[t, r]$ for $t = 0 \wedge r = s$, and
$stp_{\mathcal{A}_s}[t, r]$ for $\bigvee y_1 \bigvee y_2 \, (y_1' = t \square y_2' = r \square \langle\langle s, y_1\rangle, y_2\rangle \in \mathcal{A})$.

PROPOSITION 6.3. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(6.3i) $bas_{\mathcal{A}_s}[s', r] \Rightarrow$;

(6.3ii) $stp_{\mathcal{A}_s}[0, r] \Rightarrow$.

Proof. *Re* 6.3i.

$$\begin{array}{c} s' = 0 \Rightarrow \\ \hline s' = 0 \wedge r = s \Rightarrow \\ \hline bas_{\mathcal{A}_s}[s', r] \Rightarrow \end{array} \, .$$

Re 6.3ii.

$$\begin{array}{c} c_1' = 0 \Rightarrow \\ \hline\hline c_1' = 0, c_2' = r, \langle c_1, c_2\rangle \in \mathcal{A}_s \Rightarrow \\ \hline\hline c_1' = 0 \square c_2' = r \square \langle c_1, c_2\rangle \in \mathcal{A}_s \Rightarrow \\ \hline\hline \bigvee y_1 \bigvee y_2 \, (y_1' = 0 \square y_2' = r \square \langle y_1, y_2\rangle \in \mathcal{A}_s) \Rightarrow \end{array} \, .$$

QED

PROPOSITION 6.4. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(6.4i) $s_1 = s_2, t_1 = t_2, r_1 = r_2, \langle\langle s_1, t_1\rangle, r_1\rangle \in \mathcal{A} \Rightarrow \langle\langle s_2, t_2\rangle, r_2\rangle \in \mathcal{A}$;

(6.4ii) $\langle t, r\rangle \in \mathcal{A}_s \Leftrightarrow \langle\langle s, t\rangle, r\rangle \in \mathcal{A}$.

Proof. This is a straightforward consequence of the way $\mathcal{A}$ and $\mathcal{A}_s$ are defined. QED

PROPOSITION 6.5. *Inferences according to the following schema are* $\mathbf{L^iD}_\lambda$*-derivable:*

$$\frac{c' = r, \langle t, c\rangle \in \mathcal{A}_s, \Gamma \Rightarrow C}{stp_{\mathcal{A}_s}[t', r], \Gamma \Rightarrow C} .$$

Proof. Employ 6.4i:

$$\begin{array}{c}
c' = r, \langle\langle s, t\rangle, c\rangle \in \mathcal{A}, \Gamma \Rightarrow C \\ \hline
b = t, c' = r, \langle\langle s, b\rangle, c\rangle \in \mathcal{A}, \Gamma \Rightarrow C \\ \hline
b' = t', c' = r, \langle\langle s, b\rangle, c\rangle \in \mathcal{A}, \Gamma \Rightarrow C \\ \hline\hline
b' = t' \square c' = r \square \langle\langle s, b\rangle, c\rangle \in \mathcal{A}, \Gamma \Rightarrow C \\ \hline\hline
\bigvee y_1 \bigvee y_2\, (y_1' = t' \square y_2' = r \square \langle\langle s, y_1\rangle, y_2\rangle \in \mathcal{A}), \Gamma \Rightarrow C
\end{array} .$$

QED

DEFINITION 6.6. $s{+}t :\equiv \mathcal{A}_s[\![t]\!]$. I shall use $\mathcal{A}_s[\![t]\!]$ and $s{+}t$ interchangeably.

The first thing to establish about this definition is that it is substitutionally transparent.

PROPOSITION 6.7. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(6.7i) $s = t \Rightarrow r + s = r + t\,;$

(6.7ii) $s = t \Rightarrow s + r = t + r\,.$

Proof. Re 6.7i.

$$\begin{array}{c}
\dfrac{s = t, t = a_1' \Rightarrow s = a_1' \qquad \dfrac{b = a_2' \Rightarrow b = a_2' \qquad \langle a_1, a_2\rangle \in \mathcal{A}_r \Rightarrow \langle a_1, a_2\rangle \in \mathcal{A}_r}{b = a_2', \langle a_1, a_2\rangle \in \mathcal{A}_r \Rightarrow b = a_2' \square \langle a_1, a_2\rangle \in \mathcal{A}_r}}{s = t, t = a_1', b = a_2', \langle a_1, a_2\rangle \in \mathcal{A}_r \Rightarrow s = a_1' \square b = a_2' \square \langle a_1, a_2\rangle \in \mathcal{A}_r} \\ \hline
s = t, t = a_1', b = a_2', \langle a_1, a_2\rangle \in \mathcal{A}_r \Rightarrow stp_{\mathcal{A}_r}[s, b] \\ \hline\hline
s = t, t = a_1' \square b = a_2' \square \langle a_1, a_2\rangle \in \mathcal{A}_r \Rightarrow stp_{\mathcal{A}_r}[s, b] \\ \hline
s = t, stp_{\mathcal{A}_r}[t, b] \Rightarrow stp_{\mathcal{A}_r}[s, b]
\end{array} ;$$

$$\begin{array}{c}
\dfrac{\dfrac{\dfrac{s = t, bas_{\mathcal{A}_r}[t, b] \Rightarrow bas_{\mathcal{A}_r}[s, b]}{s = t, bas_{\mathcal{A}_r}[t, b] \Rightarrow bas_{\mathcal{A}_r}[s, b] \vee stp_{\mathcal{A}_r}[s, b]}}{s = t, bas_{\mathcal{A}_r}[t, b] \Rightarrow \langle s, b\rangle \in \mathcal{A}_r} \qquad \dfrac{\dfrac{s = t, stp_{\mathcal{A}_r}[t, b] \Rightarrow stp_{\mathcal{A}_r}[s, b]}{s = t, stp_{\mathcal{A}_r}[t, b] \Rightarrow bas_{\mathcal{A}_r}[s, b] \vee stp_{\mathcal{A}_r}[s, b]}}{s = t, stp_{\mathcal{A}_r}[t, b] \Rightarrow \langle s, b\rangle \in \mathcal{A}_r}}{s = t, bas_{\mathcal{A}_t}[s, b] \vee stp_{\mathcal{A}_r}[t, b] \Rightarrow \langle s, b\rangle \in \mathcal{A}_r} \\ \hline
s = t, \langle t, b\rangle \in \mathcal{A}_r \Rightarrow \langle s, b\rangle \in \mathcal{A}_r
\end{array} .$$

Re 6.7ii. First:

$$\dfrac{\dfrac{\dfrac{\dfrac{r=0\Rightarrow r=0 \qquad s=t,b=t\Rightarrow b=s}{s=t,r=0,b=t\Rightarrow r=0\,\square\, b=s}}{s=t,r=0,b=t\Rightarrow (r=0\,\square\,b=s)\vee \mathit{stp}_{\mathcal A}[s,r,b]}}{s=t,r=0,b=t\Rightarrow \langle\langle s,r\rangle,b\rangle\in\mathcal A}}{s=t,\,\mathit{bas}_{\mathcal A}[t,r,b]\Rightarrow\langle\langle s,r\rangle,b\rangle\in\mathcal A}\ .$$

Next:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{s=a_1\Rightarrow s=a_1 \quad r=a_2'\Rightarrow s=a_1 \quad b=a_3'\Rightarrow b=a_3'}{s=a_1,r=a_2',b=a_3'\Rightarrow s=a_1\,\square\, r=a_2'\,\square\, b=a_3'} \quad \langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A\Rightarrow\langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A}{s=a_1,r=a_2',b=a_3',\langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A\Rightarrow s=a_1\,\square\, r=a_2'\,\square\, b=a_3'\,\square\,\langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A}}{s=a_1,r=a_2',b=a_3',\langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A\Rightarrow \mathit{stp}_{\mathcal A}[s,r,b]}}{s=t,t=a_1,r=a_2',b=a_3',\langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A\Rightarrow \mathit{stp}_{\mathcal A}[s,r,b]}}{s=t,t=a_1,r=a_2',b=a_3',\langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A\Rightarrow \mathit{bas}_{\mathcal A}[s,r,b]\vee\mathit{stp}_{\mathcal A}[s,r,b]}}{s=t,t=a_1,r=a_2',b=a_3',\langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A\Rightarrow\langle\langle s,r\rangle,b\rangle\in\mathcal A}}{s=t,t=a_1\,\square\, r=a_2'\,\square\, b=a_3'\,\square\,\langle\langle a_1,a_2\rangle,a_3\rangle\in\mathcal A\Rightarrow\langle\langle s,r\rangle,b\rangle\in\mathcal A}}{s=t,\bigvee y_1\bigvee y_2\bigvee y_3\,(t=y_1\,\square\, r=y_2'\,\square\, b=y_3'\,\square\,\langle\langle y_1,y_2\rangle,y_3\rangle\in\mathcal A)\Rightarrow\langle\langle s,r\rangle,b\rangle\in\mathcal A}\ .$$

Finish as follows:

$$\dfrac{\dfrac{s=t,\,\mathit{bas}_{\mathcal A}[t,r,b]\Rightarrow\langle\langle s,r\rangle,b\rangle\in\mathcal A \qquad s=t,\,\mathit{stp}_{\mathcal A}[t,r,b]\Rightarrow\langle\langle s,r\rangle,b\rangle\in\mathcal A}{s=t,\,\mathit{bas}_{\mathcal A}[t,r,b]\vee\mathit{stp}_{\mathcal A}[t,r,b]\Rightarrow\langle\langle s,r\rangle,b\rangle\in\mathcal A}}{s=t,\langle\langle t,r\rangle,b\rangle\in\mathcal A\Rightarrow\langle\langle s,r\rangle,b\rangle\in\mathcal A}\ .$$

QED

PROPOSITION 6.8. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(6.8i) $\Rightarrow\langle 0,s\rangle\in\mathcal A_s$;

(6.8ii) $\langle t,r\rangle\in\mathcal A_s\Rightarrow\langle t',r'\rangle\in\mathcal A_s$.

Proof. Re 6.8i. Almost trivial; left to the reader.

Re 6.8ii.

$$\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow t' = t' \qquad \Rightarrow r' = r' \qquad \langle t, r\rangle \in \mathcal{A}_s \Rightarrow \langle t, r\rangle \in \mathcal{A}_s}{\langle t, r\rangle \in \mathcal{A}_s \Rightarrow t' = t' \,\Box\, r' = r' \,\Box\, \langle t, r\rangle \in \mathcal{A}_s}}{\langle t, r\rangle \in \mathcal{A}_s \Rightarrow \bigvee y_1 \bigvee y_2\, (y_1' = t' \,\Box\, y_2' = r' \,\Box\, \langle y_1, y_2\rangle \in \mathcal{A}_s)}}{\langle t, r\rangle \in \mathcal{A}_s \Rightarrow (t' = 0 \,\Box\, r' = s) \vee \bigvee y_1 \bigvee y_2\, (y_1' = t' \,\Box\, y_2' = r' \,\Box\, \langle y_1, y_2\rangle \in \mathcal{A}_s)}}{\langle t, r\rangle \in \mathcal{A}_s \Rightarrow \langle t', r'\rangle \in \mathcal{A}_s}\,.$$

QED

Proposition 6.9. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(6.9i) $\Rightarrow s + 0 = s\,;$

(6.9ii) $\Rightarrow \langle 0, s + 0\rangle \in \mathcal{A}_s\,.$

Proof. Re 6.9i. Employ 6.8i and 6.3ii:[23]

$$\dfrac{\dfrac{\dfrac{\dfrac{\Rightarrow \langle 0, s\rangle \in \mathcal{A}_s \qquad a \in s \Rightarrow a \in s}{\langle 0, s\rangle \in \mathcal{A}_s \to a \in s \Rightarrow a \in s}}{\bigwedge y\, (\langle 0, y\rangle \in \mathcal{A}_s \to a \in y) \Rightarrow a \in s}}{a \in (s + 0) \Rightarrow a \in s} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{a \in s, s = b \Rightarrow a \in b}{a \in s, 0 = 0 \wedge s = b \Rightarrow a \in b} \qquad \mathit{stp}_{\mathcal{A}_s}[0, b] \Rightarrow}{a \in s, \mathit{bas}_{\mathcal{A}_s}[0, b] \vee \mathit{stp}_{\mathcal{A}_s}[0, b] \Rightarrow a \in b}}{a \in s, \langle 0, b\rangle \in \mathcal{A}_s \Rightarrow a \in b}}{a \in s \Rightarrow \langle 0, b\rangle \in \mathcal{A}_s \to a \in b}}{a \in s \Rightarrow \bigwedge y\, (\langle 0, y\rangle \in \mathcal{A}_s \to a \in y)}}{a \in s \Rightarrow a \in (s + 0)}}{\Rightarrow s + 0 = s}\,.$$

Re 6.9ii. Employ 6.9i:

$$\dfrac{\dfrac{\dfrac{\Rightarrow 0 = 0 \qquad \Rightarrow s = s + 0}{\Rightarrow 0 = 0 \wedge s = s + 0}}{\Rightarrow (0 = 0 \wedge s = s + 0) \vee \bigvee y_1 \bigvee y_2\, (y_1' = 0 \,\Box\, y_2' = b \,\Box\, \langle y_1, y_2\rangle \in \mathcal{A}_s)}}{\Rightarrow \langle 0, s + 0\rangle \in \mathcal{A}_s}\,.$$

QED

[23] Note that due to the fixed point definition of addition no **Z**-inference is needed here, in contrast to the classical approach as pursued in [21], p. 1889: proposition 137.13 requires an inference according to proposition 137.10 on p. 1887, and thereby a **Z**-inference.

Proposition 6.10. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(6.10i) $\langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t') \Rightarrow a \in (s+t)'$;

(6.10ii) $uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)' \Rightarrow a \in (s+t')$;

(6.10iii) $uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s \Rightarrow s+t' = (s+t)'$;

(6.10iv) $uni[t, \mathcal{A}_s], 2[\langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow \langle t', s+t'\rangle \in \mathcal{A}_s$.

Proof. Re 6.10i. As usual, this direction is almost trivial in view of how application is defined. Employ 6.8ii:

$$\begin{array}{c}
\langle t, s+t\rangle \in \mathcal{A}_s \Rightarrow \langle t, (s+t)'\rangle \in \mathcal{A}_s \qquad a \in (s+t)' \Rightarrow a \in (s+t)' \\
\hline
\langle t, s+t\rangle \in \mathcal{A}_s, \langle t, (s+t)'\rangle \in \mathcal{A}_s \rightarrow a \in (s+t)' \Rightarrow a \in (s+t)' \\
\hline
\langle t, s+t\rangle \in \mathcal{A}_s, \bigwedge y (\langle t, y\rangle \in \mathcal{A}_s \rightarrow a \in y) \Rightarrow a \in (s+t)' \\
\hline
\langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t') \Rightarrow a \in (s+t)'
\end{array} .$$

Re 6.10ii. This is the direction which requires uniqueness. Employ 6.11iv:

$$\begin{array}{c}
\langle t, s+t\rangle \in \mathcal{A}_s, \langle t, c_2\rangle \in \mathcal{A}_s \Rightarrow \langle t, s+t\rangle \in \mathcal{A}_s \Box \langle t, c_2\rangle \in \mathcal{A}_s \quad s+t = c_2, a \in (s+t)' \Rightarrow a \in c_2' \\
\hline
\langle t, s+t\rangle \in \mathcal{A}_s \Box \langle t, c_2\rangle \in \mathcal{A}_s \rightarrow s+t = c_2, \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)', \langle t, c_2\rangle \in \mathcal{A}_s \Rightarrow a \in c_2' \\
\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)', \langle t, c_2\rangle \in \mathcal{A}_s \Rightarrow a \in c_2' \\
\hline\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)', c_1 = t, c_2' = b, \langle c_1, c_2\rangle \in \mathcal{A}_s \Rightarrow a \in b \\
\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)', c_1' = t', c_2' = b, \langle c_1, c_2\rangle \in \mathcal{A}_s \Rightarrow a \in b \\
\hline\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)', c_1' = t' \Box c_2' = b \Box \langle c_1, c_2\rangle \in \mathcal{A}_s \Rightarrow a \in b \\
\hline\hline
bas_{\mathcal{A}_s}[t', b] \Rightarrow \qquad uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)', stp_{\mathcal{A}_s}[t', b] \Rightarrow a \in b \\
\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)', bas_{\mathcal{A}_s}[t', b] \vee stp_{\mathcal{A}_s}[t', b] \Rightarrow a \in b \\
\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)', \langle t', b\rangle \in \mathcal{A}_s \Rightarrow a \in b \\
\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)' \Rightarrow \langle t', b\rangle \in \mathcal{A}_s \rightarrow a \in b \\
\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)' \Rightarrow \bigwedge y (\langle t', y\rangle \in \mathcal{A}_s \rightarrow a \in y) \\
\hline
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)' \Rightarrow a \in (s+t')
\end{array} .$$

Re 6.10iii. Employ 6.10i and 6.10ii:

$$\frac{\langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t') \Rightarrow a \in (s+t)' \quad uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s, a \in (s+t)' \Rightarrow a \in (s+t')}{uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s \Rightarrow s+t' = (s+t)'}\,.$$

Re 6.10iv. Employ 6.10iii:

$$\begin{array}{c}
uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s \Rightarrow (s+t)' = s+t' \quad \langle t, s+t\rangle \in \mathcal{A}_s \Rightarrow \langle t, s+t\rangle \in \mathcal{A}_s \\ \hline
\Rightarrow t' = t' \quad uni[t, \mathcal{A}_s], 2[\langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow (s+t)' = s+t' \,\square\, \langle t, s+t\rangle \in \mathcal{A}_s \\ \hline
uni[t, \mathcal{A}_s], 2[\langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow t' = t' \,\square\, (s+t)' = s+t' \,\square\, \langle t, s+t\rangle \in \mathcal{A}_s \\ \hline\hline
uni[t, \mathcal{A}_s], 2[\langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow \bigvee y_1 \bigvee y_2\, (y_1' = t' \,\square\, y_2' = s+t' \,\square\, \langle y_1, y_2\rangle \in \mathcal{A}_s) \\ \hline
uni[t, \mathcal{A}_s], 2[\langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow bas_{\mathcal{A}_s}[t', s+t'] \vee stp_{\mathcal{A}_s}[t', s+t'] \\ \hline
uni[t, \mathcal{A}_s], 2[\langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow \langle t', s+t'\rangle \in \mathcal{A}_s
\end{array}\,.$$

QED

Proposition 6.11. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(6.11i) $\langle 0, r\rangle \in \mathcal{A}_s \Rightarrow s = r\,;$

(6.11ii) $\Rightarrow uni[0, \mathcal{A}_s]\,;$

(6.11iii) $uni[t, \mathcal{A}_s], stp_{\mathcal{A}_s}[s, a', c_1], stp_{\mathcal{A}_s}[s, a', c_2] \Rightarrow c_1 = c_2\,;$

(6.11iv) $uni[t, \mathcal{A}_s] \Rightarrow uni[t', \mathcal{A}_s]\,.$

Proof. *Re* 6.11i. Almost trivial, but nevertheless .... Employ 6.3ii:

$$\begin{array}{c}
\dfrac{s = r \Rightarrow s = r}{0 = 0 \wedge s = r \Rightarrow s = r} \quad \bigvee y_1 \bigvee y_2 (y_1' = 0 \,\square\, y_2' = r \,\square\, \langle y_1, y_2\rangle \in \mathcal{A}_s) \Rightarrow s = r \\ \hline
(0 = 0 \,\square\, r = s) \vee \bigvee y_1 \bigvee y_2 (y_1' = 0 \,\square\, y_2' = r \,\square\, \langle y_1, y_2\rangle \in \mathcal{A}_s) \Rightarrow s = r \\ \hline
\langle 0, r\rangle \in \mathcal{A}_s \Rightarrow s = r
\end{array}\,.$$

Re 6.11ii. Variation of 6.11i; left to the reader.
Re 6.11iii. The essential step is that of the 'reversibility' of the successor relation, in the sense of 4.3vii, which is being applied twice in the following deduction. This is where the new definition of the successor with 2.2 above comes in:

$$
\begin{array}{c}
\langle a,b_2\rangle\in\mathcal{A}_s,\langle a,b_1\rangle\in\mathcal{A}_s\Rightarrow\langle a,b_1\rangle\in\mathcal{A}_s\,\square\,\langle a,b_2\rangle\in\mathcal{A}_s \qquad b_1=b_2\Rightarrow b_2'=b_1'\\
\hline
\langle a,b_1\rangle\in\mathcal{A}_s\,\square\,\langle t,b_2\rangle\in\mathcal{A}_s\rightarrow b_1=b_2,\langle t,b_2\rangle\in\mathcal{A}_s,\langle t,b_1\rangle\in\mathcal{A}_s\Rightarrow b_2'=b_1'\\
\hline
\mathit{uni}[t,\mathcal{A}_s],\langle t,b_2\rangle\in\mathcal{A}_s,\langle t,b_1\rangle\in\mathcal{A}_s\Rightarrow b_2'=b_1'\\
\hline\hline
\mathit{uni}[t,\mathcal{A}_s],\langle t,b_2\rangle\in\mathcal{A}_s,a_1=t,b_1'=c_2,\langle a_1,b_1\rangle\in\mathcal{A}_s\Rightarrow b_2'=c_2\\
\hline
\mathit{uni}[t,\mathcal{A}_s],\langle t,b_2\rangle\in\mathcal{A}_s,a_1'=t',b_1'=c_2,\langle a_1,b_1\rangle\in\mathcal{A}_s\Rightarrow b_2'=c_2\\
\hline\hline
\mathit{uni}[t,\mathcal{A}_s],\langle t,b_2\rangle\in\mathcal{A}_s,\mathit{stp}_{\mathcal{A}_s}[t',c_2]\Rightarrow b_2'=c_2\\
\hline\hline
\mathit{uni}[t,\mathcal{A}_s],b_1=t,b_2'=c_1,\langle b_1,b_2\rangle\in\mathcal{A}_s,\mathit{stp}_{\mathcal{A}_s}[t',c_2]\Rightarrow c_1=c_2\\
\hline
\mathit{uni}[t,\mathcal{A}_s],b_1'=t',b_2'=c_1,\langle b_1,b_2\rangle\in\mathcal{A}_s,\mathit{stp}_{\mathcal{A}_s}[t',c_2]\Rightarrow c_1=c_2\\
\hline\hline
\mathit{uni}[t,\mathcal{A}_s],b_1'=t'\,\square\,b_2'=c_1\,\square\,\langle b_1,b_2\rangle\in\mathcal{A}_s,\mathit{stp}_{\mathcal{A}_s}[t',c_2]\Rightarrow c_1=c_2\\
\hline\hline
\mathit{uni}[t,\mathcal{A}_s],\mathit{stp}_{\mathcal{A}_s}[t',c_1],\mathit{stp}_{\mathcal{A}_s}[t',c_2]\Rightarrow c_1=c_2
\end{array}\ .
$$

Re 6.11iv. Employ 6.11iii:

$$
\begin{array}{c}
\mathit{bas}_{\mathcal{A}_s}[t',c_2]\Rightarrow \quad \mathit{uni}[t,\mathcal{A}_s],\mathit{stp}_{\mathcal{A}_s}[t',c_1],\mathit{stp}_{\mathcal{A}_s}[t',c_2]\Rightarrow c_1=c_2\\
\hline
\mathit{bas}_{\mathcal{A}_s}[t',c_1]\Rightarrow \quad \mathit{uni}[t,\mathcal{A}_s],\mathit{stp}_{\mathcal{A}_s}[t',c_1],\mathit{bas}_{\mathcal{A}_s}[t',c_2]\vee\mathit{stp}_{\mathcal{A}_s}[t',c_2]\Rightarrow c_1=c_2\\
\hline
\mathit{uni}[t,\mathcal{A}_s],\mathit{bas}_{\mathcal{A}_s}[t',c_1]\vee\mathit{stp}_{\mathcal{A}_s}[t',c_1],\mathit{bas}_{\mathcal{A}_s}[t',c_2]\vee\mathit{stp}_{\mathcal{A}_s}[t',c_2]\Rightarrow c_1=c_2\\
\hline\hline
\mathit{uni}[t,\mathcal{A}_s],\langle t',c_1\rangle\in\mathcal{A}_s,\langle t',c_2\rangle\in\mathcal{A}_s\Rightarrow c_1=c_2\\
\hline
\mathit{uni}[t,\mathcal{A}_s]\Rightarrow\mathit{uni}[t',\mathcal{A}_s]
\end{array}\ .
$$

QED

REMARK 6.12. Two separate inductions would now do the job; a first one (according to 4.7i) to yield $t\in\mathbf{N}^{\circ}\Rightarrow \mathit{uni}[s,t,\mathcal{A}]$:

1. $\Rightarrow \mathit{uni}[0,\mathcal{A}_s]$ 6.11ii
2. $\mathit{uni}[b,\mathcal{A}_s]\Rightarrow\mathit{uni}[b',\mathcal{A}_s]$ 6.11iv

and a second one (according to 4.7vii, because of the double occurrence of the antecedent formula) to yield $t\in\mathbf{N}^{\circ}\Rightarrow\langle\langle s,t\rangle,s+t]\!]\rangle\in\mathcal{A}$:

3. $\Rightarrow\langle 0,s+0\rangle\in\mathcal{A}_s$ 6.9ii
5. $b\in\mathbf{N}^{\circ},2[\langle b,s+b\rangle\in\mathcal{A}_s]\Rightarrow\langle b',s+b'\rangle\in\mathcal{A}$

where the last one is obtained from 6.10iv

$$
\frac{b\in\mathbf{N}^{\circ}\Rightarrow\mathit{uni}[s,b,\mathcal{A}]\qquad \mathit{uni}[s,b,\mathcal{A}],2[\langle b,s+b\rangle\in\mathcal{A}_s]\Rightarrow\langle b',s+b'\rangle\in\mathcal{A}}{b\in\mathbf{N}^{\circ},2[\langle b,s+b\rangle\in\mathcal{A}_s]\Rightarrow\langle b',s+b'\rangle\in\mathcal{A}}\ \text{cut}\ .
$$

Altogether, this is not the most economical way to obtain the recursion equations for addition and this is why I combine the two inductions into one which saves considerably on **Z**-inferences.

PROPOSITION 6.13. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(6.13i) $\Rightarrow uni[0, \mathcal{A}_s] \sqcap \langle 0, s+0\rangle \in \mathcal{A}_s$;

(6.13ii) $2[uni[t, \mathcal{A}_s] \sqcap \langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow uni[t', \mathcal{A}_s] \sqcap \langle t', s+t'\rangle \in \mathcal{A}_s$.

Proof. *Re* 6.13i. Employ 6.11ii and 6.9ii:

$$\frac{\Rightarrow uni[0, \mathcal{A}_s] \qquad \Rightarrow \langle 0, s+0\rangle \in \mathcal{A}_s}{\Rightarrow uni[0, \mathcal{A}_s] \sqcap \langle 0, s+0\rangle \in \mathcal{A}_s} \ .$$

Re 6.13ii. Employ 6.11iv and 6.10iv:

$$\dfrac{\dfrac{uni[t, \mathcal{A}_s] \Rightarrow uni[t', \mathcal{A}_s] \qquad uni[t, \mathcal{A}_s], 2[\langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow \langle t', s+t'\rangle \in \mathcal{A}_s}{2[uni[t, \mathcal{A}_s]], 2[\langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow uni[t', \mathcal{A}_s] \sqcap \langle t', s+t'\rangle \in \mathcal{A}_s}}{2[uni[t, \mathcal{A}_s] \sqcap \langle t, s+t\rangle \in \mathcal{A}_s] \Rightarrow uni[t', \mathcal{A}_s] \sqcap \langle t', s+t'\rangle \in \mathcal{A}_s} \ .$$

QED

Everything so far has been $\mathbf{L^iD}_\lambda$-deducible. Now come the final steps, the ones that involve **Z**-inferences, be that in the form of a "modal" inference or an induction.

PROPOSITION 6.14. $\mathbf{L^iD}_\lambda^{Z\restriction 4} \vdash \ t \in \mathbf{N}^\circ \Rightarrow s + t' = (s+t)'$.

Proof. Employ 6.13i and 6.13ii with an inference according to schema 4.7vii. QED

REMARKS 6.15. (1) One last time I want to spell out an induction in terms of higher order logic, *i.e.*, $\mathbf{N}^\circ$. Employ 6.13ii, 6.13i and 6.10iii:

$$\dfrac{\dfrac{\dfrac{\dfrac{\begin{matrix}\mathbf{L^iD}_\lambda^{Z\restriction 2}\\ \vdots \\ b\in\xi \Rightarrow b'\in\xi\end{matrix}}{\Rightarrow b\in\xi \to b'\in\xi}}{\Rightarrow \bigwedge z(z\in\xi \to z'\in\xi)} \qquad \dfrac{\begin{matrix}\mathbf{L^iD}_\lambda^{Z\restriction 1}\\ \vdots \\ \Rightarrow 0\in\xi\end{matrix} \qquad \dfrac{\dfrac{\dfrac{uni[t, \mathcal{A}_s], \langle t, s+t\rangle \in \mathcal{A}_s \Rightarrow s+t' = (s+t)'}{uni[t, \mathcal{A}_s] \sqcap \langle t, s+t\rangle \in \mathcal{A}_s \Rightarrow s+t' = (s+t)'}}{\Box(uni[t, \mathcal{A}_s] \sqcap \langle t, s+t\rangle \in \mathcal{A}_s) \Rightarrow s+t' = (s+t)'}}{t\in\xi \Rightarrow s+t' = (s+t)'}}{0\in\xi \to t\in\xi \Rightarrow s+t' = (s+t)'}}{\bigwedge z((z\in\xi) \to (z'\in\xi)) \supset ((0\in\xi) \to (t\in\xi)) \Rightarrow s+t' = (s+t)'}\,{}^{+1}}{t\in\mathbf{N}^\circ \Rightarrow s+t' = (s+t)'} \ ,$$

where $\xi :\equiv \lambda x \,\square\, (uni[x, \mathcal{A}_s] \,\square\, \langle x, s + x\rangle \in \mathcal{A}_s)$.
(2) It doesn't need much to see that the foregoing approach to proving the recursion equations for addition can be extended to all primitive recursive functions, *i.e.*, the recursion equations for all primitive recursive function are provable in $\mathbf{L^iD}_{\lambda}^{Z\restriction 4}$. The approach to establishing the recursion equations for addition can be divided into four blocks:

(1) $\Rightarrow \langle 0, s\rangle \in \mathcal{A}_s\,;$
$\langle t, r\rangle \in \mathcal{A}_s \Rightarrow \langle t', r'\rangle \in \mathcal{A}_s\,.$

(2) $\Rightarrow uni[0, \mathcal{A}_s]\,;$
$uni[t, \mathcal{A}_s] \Rightarrow uni[t', \mathcal{A}_s]\,.$

(3) $uni[t, \mathcal{A}_s], \langle t, \mathcal{A}_s[\![t]\!]\rangle \in \mathcal{A}_s \Rightarrow \mathcal{A}_s[\![t']\!] = \mathcal{A}_s[\![t]\!]'\,;$

(4) $\Rightarrow uni[0, \mathcal{A}_s[\![0]\!]]\,;$
$uni[t, \mathcal{A}_s], 2\,[\langle t, \mathcal{A}_s[\![t]\!]\rangle \in \mathcal{A}_s] \Rightarrow \langle t', \mathcal{A}_s[\![t']\!]\rangle \in \mathcal{A}_s\,.$

This approach fits to all primitive-recursive functions. If h_s is a one-place primitive recursive function defined in terms of another one-place function f and a two-place function g_s as the fixed point

$$h_s = \lambda x_1 x_2 \,(x_1 = 0 \wedge x_2 = f[\![s]\!]) \vee \bigvee y_1 \bigvee y_2 \,(y_1' = x_1 \,\square\, g_s[\![s, y_2]\!] = x_2 \,\square\, \langle y_1, y_2\rangle \in h_s)\,,$$

then the essential ingredients for obtaining the recursion equations are:

(1) $\Rightarrow \langle 0, f[\![s]\!]\rangle \in h_s\,;$
$\langle t, r\rangle \in h_s \Rightarrow \langle t', g_s[\![t, r]\!]\rangle \in h_s\,.$

(2) $\Rightarrow uni[0, h_s]\,;$
$uni[t, h_s] \Rightarrow uni[t', h_s]\,.$

(3) $uni[t, h_s], \langle t, h_s[\![t]\!]\rangle \in h_s \Rightarrow h_s[\![t']\!] = g_s[\![t, h_s[\![t]\!]]\!]\,.$

(4) $\Rightarrow uni[0, h_s[\![0]\!]]\,;$
$uni[t, h_s], 2\,[\langle t, h_s[\![t]\!]\rangle \in h_s] \Rightarrow \langle t', h_s[\![t']\!]\rangle \in h_s\,.$

I leave it at these hints trusting that they are sufficient to support my claim that the approach extends to all primitive-recursive functions.

7. Totality of addition

As with the predecessor function, totality can't hold for addition in the sense that it does for the successor operation: $s+t$ just won't be in $\mathbf{N}^\circ$, no matter what its numerical value. But that's what the notion of $\mathbf{N}$ (definition 5.3 above) has been introduced for: if $s\in\mathbf{N}$ and $t\in\mathbf{N}$ then $(s+t)\in\mathbf{N}$.

REMARK 7.1. It should be clear that the totality of addition can be established on the basis of the recursion equations as obtained in 6.9i and 6.14 above employing just another simple induction:

$$\dfrac{\dfrac{\dfrac{s\in\mathbf{N}^\circ \Rightarrow s\in\mathbf{N}^\circ \qquad \Rightarrow s = s+0}{s\in\mathbf{N}^\circ \Rightarrow s\in\mathbf{N}^\circ \,\Box\, s = s+0}}{s\in\mathbf{N}^\circ \Rightarrow (s+0)\in\mathbf{N}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{c\in\mathbf{N}^\circ \Rightarrow c'\in\mathbf{N}^\circ \quad \dfrac{b\in\mathbf{N}^\circ \Rightarrow (s+b)' = s+b'}{b\in\mathbf{N}^\circ, c = s+b \Rightarrow c' = s+b'}}{b\in\mathbf{N}^\circ, c\in\mathbf{N}^\circ, c = s+b \Rightarrow c'\in\mathbf{N}^\circ \,\Box\, c' = s+b'}}{b\in\mathbf{N}^\circ, c\in\mathbf{N}^\circ \,\Box\, c = s+b \Rightarrow c'\in\mathbf{N}^\circ \,\Box\, c' = s+b'}}{b\in\mathbf{N}^\circ, c\in\mathbf{N}^\circ \,\Box\, c = s+b \Rightarrow (s+b')\in\mathbf{N}}}{b\in\mathbf{N}^\circ, (s+b)\in\mathbf{N} \Rightarrow (s+b')\in\mathbf{N}}}{s\in\mathbf{N}^\circ, t\in\mathbf{N}^\circ \Rightarrow (s+t)\in\mathbf{N}}\ .$$

And then use this for a cut in the inference marked † below:

$$\dfrac{\dfrac{\dfrac{s\in\mathbf{N}^\circ, t\in\mathbf{N}^\circ \Rightarrow (s+t)\in\mathbf{N} \qquad \dfrac{\dfrac{\dfrac{\dfrac{c\in\mathbf{N}^\circ \Rightarrow c\in\mathbf{N}^\circ \quad \dfrac{c = a+b \Rightarrow c = a+b}{a = s, b = t, c = s+b \Rightarrow c = s+t}}{c\in\mathbf{N}^\circ, a = s, b = t, c = s+b \Rightarrow c\in\mathbf{N}^\circ \,\Box\, c = s+t}}{a = s, b = t, c\in\mathbf{N}^\circ \,\Box\, c = s+b \Rightarrow c\in\mathbf{N}^\circ \,\Box\, c = s+t}}{a = s, b = t, c\in\mathbf{N}^\circ \,\Box\, c = a+b \Rightarrow (s+t)\in\mathbf{N}}}{a = s, b = t, (a+b)\in\mathbf{N} \Rightarrow (s+t)\in\mathbf{N}}}{a\in\mathbf{N}^\circ, b\in\mathbf{N}^\circ, a = s, b = t \Rightarrow (s+t)\in\mathbf{N}}\,\dagger}{a\in\mathbf{N}^\circ \,\Box\, a = s, b\in\mathbf{N}^\circ \,\Box\, b = t \Rightarrow (s+t)\in\mathbf{N}}}{s\in\mathbf{N}, t\in\mathbf{N} \Rightarrow (s+t)\in\mathbf{N}}\ .$$

The point is to get around this cut and the additional induction.

Proposition 7.2. *The following is* $\mathbf{L^{i}D}_{\lambda}$*-deducible:*

(7.2i) $s\in\mathbf{N}^{\circ} \Rightarrow (s+0)\in\mathbf{N}\,;$

(7.2ii) $uni[t,\mathcal{A}_s], \langle t,\mathcal{A}_s[\![t]\!]\rangle\in\mathcal{A}_s, (s+t)\in\mathbf{N}) \Rightarrow (s+t')\in\mathbf{N}\,.$

Proof. *Re* 7.2i. Employ 6.9i:

$$\begin{array}{c}
s\in\mathbf{N}^{\circ} \Rightarrow s\in\mathbf{N}^{\circ} \qquad \Rightarrow (s+0)=s \\ \hline
s\in\mathbf{N}^{\circ} \Rightarrow s\in\mathbf{N}^{\circ} \square (s+0)=s \\ \hline
s\in\mathbf{N}^{\circ} \Rightarrow \bigvee y(y\in\mathbf{N}^{\circ} \square (s+0)=y) \\ \hline
s\in\mathbf{N}^{\circ} \Rightarrow (s+0)\in\mathbf{N}
\end{array}\;.$$

Re 7.2ii. Employ 6.14:

$$\begin{array}{c}
b\in\mathbf{N}^{\circ} \Rightarrow b'\in\mathbf{N}^{\circ} \qquad \begin{array}{c} uni[t,\mathcal{A}_s], \langle t,s+t\rangle\in\mathcal{A}_s \Rightarrow s+t'=(s+t)' \\ \hline uni[t,\mathcal{A}_s], \langle t,s+t\rangle\in\mathcal{A}_s, s+t=b \Rightarrow s+t'=b' \end{array} \\ \hline
uni[t,\mathcal{A}_s], \langle t,s+t\rangle\in\mathcal{A}_s, b\in\mathbf{N}^{\circ}, s+t=b \Rightarrow b'\in\mathbf{N}^{\circ} \square s+t'=b' \\ \hline
uni[t,\mathcal{A}_s], \langle t,s+t\rangle\in\mathcal{A}_s, b\in\mathbf{N}^{\circ}, s+t=b \Rightarrow \bigvee y\,(y\in\mathbf{N}^{\circ} \square s+t'=y) \\ \hline
t\in\mathbf{N}^{\circ}, b\in\mathbf{N}^{\circ}, s+t=b \Rightarrow (s+t')\in\mathbf{N} \\ \hline
uni[t,\mathcal{A}_s], \langle t,s+t\rangle\in\mathcal{A}_s, b\in\mathbf{N}^{\circ} \square s+t=b \Rightarrow (s+t')\in\mathbf{N} \\ \hline
uni[t,\mathcal{A}_s], \langle t,s+t\rangle\in\mathcal{A}_s, \bigvee y\,(y\in\mathbf{N}^{\circ} \square s+t=y) \Rightarrow (s+t')\in\mathbf{N} \\ \hline
uni[t,\mathcal{A}_s], \langle t,s+t\rangle\in\mathcal{A}_s, (s+t)\in\mathbf{N} \Rightarrow (s+t')\in\mathbf{N}
\end{array}\;.$$

QED

Proposition 7.3. *The following is* $\mathbf{L^{i}D}_{\lambda}$*-deducible:*

(7.3i) $s\in\mathbf{N}^{\circ} \Rightarrow uni[0,\mathcal{A}_s] \square \langle 0,s+0\rangle\in\mathcal{A}_s \square (s+0)\in\mathbf{N}\,;$

(7.3ii) $3\,[uni[t,\mathcal{A}_s] \square \langle t,s+t\rangle\in\mathcal{A}_s \square (s+t)\in\mathbf{N}] \Rightarrow;$

$uni[t',\mathcal{A}_s] \square \langle t',s+t'\rangle\in\mathcal{A}_s \square (s+t')\in\mathbf{N}\,;$

(7.3iii) $uni[b,\mathcal{A}_a] \square \langle b,a+b\rangle\in\mathcal{A}_a \square (a+b)\in\mathbf{N}), s=a, t=b \Rightarrow$

$(s+t)\in\mathbf{N}\,.$

Proof. *Re* 7.3i. Conjunction of 6.13i and 7.2i.
Re 7.3ii. Conjunction of 6.13ii and 7.2ii.

Re 7.3iii. Employ 6.7i and 6.7ii:

$$
\dfrac{c\in\mathbf{N}^{\circ} \Rightarrow c\in\mathbf{N}^{\circ} \qquad \dfrac{s=a, t=b \Rightarrow (s+t)=(a+b)}{s=a, t=b, (a+b)=c \Rightarrow (s+t)=c}}{c\in\mathbf{N}^{\circ}, s=a, t=b, (a+b)=c \Rightarrow c\in\mathbf{N}^{\circ} \square (s+t)=c}
$$

$$
\begin{array}{c}
c\in\mathbf{N}^{\circ}, s=a, t=b, (a+b)=c \Rightarrow c\in\mathbf{N}^{\circ} \square (s+t)=c \\
\hline
c\in\mathbf{N}^{\circ}, s=a, t=b, (a+b)=c \Rightarrow \bigvee y\,(y\in\mathbf{N}^{\circ} \square (s+t)=y) \\
\hline
c\in\mathbf{N}^{\circ}, s=a, t=b, (a+b)=c \Rightarrow (s+t)\in\mathbf{N} \\
\hline
c\in\mathbf{N}^{\circ} \square (a+b)=c, s=a, t=b \Rightarrow (s+t)\in\mathbf{N} \\
\hline
\bigvee y\,(y\in\mathbf{N}^{\circ} \square (a+b)=y), s=a, t=b \Rightarrow (s+t)\in\mathbf{N} \\
\hline
(a+b)\in\mathbf{N}, s=a, t=b \Rightarrow (s+t)\in\mathbf{N} \\
\hline\hline
uni[b,\mathcal{A}_a], \langle b, a+b\rangle\in\mathcal{A}_a, (a+b)\in\mathbf{N}, s=a, t=b \Rightarrow (s+t)\in\mathbf{N} \\
\hline\hline
uni[b,\mathcal{A}_a] \square \langle b, a+b\rangle\in\mathcal{A}_a \square (a+b)\in\mathbf{N}, s=a, t=b \Rightarrow (s+t)\in\mathbf{N}
\end{array}\ .
$$

QED

PROPOSITION 7.4. $\mathbf{L^{i}D}_{\lambda}^{Z\restriction 9} \vdash\ s\in\mathbf{N}, t\in\mathbf{N} \Rightarrow (s+t)\in\mathbf{N}$.

Proof. This is an immediate consequence of 7.3i, 7.3ii, and 7.3iii by means of 4.7viii. QED

8. Multiplication

The schema of the foregoing two sections will now be applied to multiplication. In view of the similarity of the approach, I go fairly quickly through the relevant steps.

PROPOSITION 8.1. *There exists a term $\mathcal{M}$ such that:*

$$
\begin{aligned}
&\mathbf{L^{i}D}_{\lambda} \vdash \mathcal{M} = \lambda x_1 x_2 x_3\,((x_2 = 0 \wedge x_3 = 0) \vee \\
&\quad \bigvee y_1 \bigvee y_2 \bigvee y_2\,(y_1 = x_1 \square y_2' = x_2 \square y_3 + y_1 = x_3 \square \langle\langle y_1, y_2\rangle, y_3\rangle\in\mathcal{M}))\,.
\end{aligned}
$$

Proof. As usual, this is an immediate consequence of the fixed-point property. QED

CONVENTIONS 8.2. (1) As in the case of addition I introduce an abbreviation to simplify presentation:

$$\mathcal{M}_s :\equiv \lambda x_2 x_3\,((x_2 = 0 \wedge x_3 = 0) \vee \\ \bigvee y_2 \bigvee y_3\,(y_2' = x_2 \,\Box\, y_3 + y_1 = x_3 \,\Box\, \langle\langle s, y_2\rangle, y_3\rangle \in \mathcal{M}))\,.$$

As before, the full definition is only really needed in the proof of one of the substitution properties below.
(2) In order to save space in presentations, I shall occasionally use the following abbreviations:

$bas_{\mathcal{M}_s}[t,r]$ for $t = 0 \wedge r = 0$, and
$stp_{\mathcal{M}_s}[t,r]$ for $\bigvee y_2 \bigvee y_3\,(y_2' = t \,\Box\, y_2 + s = r \,\Box\, \langle\langle s, y_2\rangle, y_3\rangle \in \mathcal{M})$.

PROPOSITION 8.3. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(8.3i) $bas_{\mathcal{M}_s}[s', r] \Rightarrow$;

(8.3ii) $stp_{\mathcal{M}_s}[0, r] \Rightarrow$.

Proof. As for 6.3i and 6.3ii; left to the reader. QED

PROPOSITION 8.4. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(8.4i) $s_1 = s_2, t_1 = t_2, r_1 = r_2, \langle\langle s_1, t_1\rangle, r_1\rangle \in \mathcal{M} \Rightarrow \langle\langle s_2, t_2\rangle, r_2\rangle \in \mathcal{M}$;

(8.4ii) $\Rightarrow \langle t, r\rangle \in \mathcal{M}_s \Leftrightarrow \langle\langle s, t\rangle, r\rangle \in \mathcal{M}$.

Proof. As for 6.7. Left to the reader. QED

PROPOSITION 8.5. *Inferences according to the following schema are* $\mathbf{L^iD}_\lambda$*-derivable:*

$$\frac{r_2 = r_1 + s, \langle t, r_1\rangle \in \mathcal{M}_s, \Gamma \Rightarrow C}{stp_{\mathcal{M}_s}[t', r_2], \Gamma \Rightarrow C}\,.$$

Proof. As for 6.5; left to the reader. QED

DEFINITION 8.6. $s \cdot t :\equiv \mathcal{M}_s[\![t]\!]$. I shall use $\mathcal{M}_s[\![t]\!]$ and $(s \cdot t)$ interchangeably.

As for the case of addition, the first thing to establish about this definition is that it is substitutionally transparent.

PROPOSITION 8.7. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(8.7i) $s = t \Rightarrow s \cdot r = t \cdot r$;

(8.7ii) $s = t \Rightarrow r \cdot s = r \cdot t$.

Proof. As for 6.7. Left to the reader. QED

PROPOSITION 8.8. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(8.8i) $\Rightarrow \langle 0, 0 \rangle \in \mathcal{M}_s$;

(8.8ii) $\langle t, r \rangle \in \mathcal{M}_s \Rightarrow \langle t', r + s \rangle \in \mathcal{M}_s$.

Proof. As for 6.8; left to the reader. QED

PROPOSITION 8.9. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(8.9i) $\Rightarrow s \cdot 0 = 0$;

(8.9ii) $\Rightarrow \langle 0, s \cdot 0 \rangle \in \mathcal{M}_s$.

Proof. As for 6.9; left to the reader. QED

PROPOSITION 8.10. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(8.10i) $\langle t, s \cdot t \rangle \in \mathcal{M}_s, a \in (s \cdot t') \Rightarrow a \in (s \cdot t + s)$;

(8.10ii) $uni[t, \mathcal{M}_s], \langle t, s \cdot t \rangle \in \mathcal{M}_s, a \in (s \cdot t + s) \Rightarrow a \in (s \cdot t')$;

(8.10iii) $uni[t, \mathcal{M}_s], \langle t, s \cdot t \rangle \in \mathcal{M}_s \Rightarrow s \cdot t' = s \cdot t + s$;

(8.10iv) $uni[t, \mathcal{M}_s], 2\,[\langle t, s \cdot t \rangle \in \mathcal{M}_s] \Rightarrow \langle t', s \cdot t' \rangle \in \mathcal{M}_s$.

Proof. Re 8.10i. This follows directly from 8.8ii in the usual way.
Re 8.10ii. For the nonce, let $\mathfrak{M}[s, t, c]$ stand for $\langle t, s \cdot t \rangle \in \mathcal{M}_s \sqcap \langle t, c \rangle \in \mathcal{M}_s$:

$$
\begin{array}{c}
\dfrac{\langle t, s\cdot t\rangle\in\mathcal{M}_s, \langle t, c\rangle\in\mathcal{M}_s \Rightarrow \mathfrak{M}[s,t,c] \qquad \dfrac{a\in(c+s)\Rightarrow a\in(c+s)}{s\cdot t = c, a\in(s\cdot t+s)\Rightarrow a\in(c+s)}}{\mathfrak{M}[s,t,c]\rightarrow s\cdot t = c, \langle t, s\cdot t\rangle\in\mathcal{M}_s, a\in(s\cdot t+s), \langle t, c\rangle\in\mathcal{M}_s \Rightarrow a\in(c+s)} \\
\hline
uni[s,t,\mathcal{M}], \langle t, s\cdot t\rangle\in\mathcal{M}_s, a\in(s\cdot t+s), \langle t, c\rangle\in\mathcal{M}_s \Rightarrow a\in(c+s) \\
\hline
uni[s,t,\mathcal{M}], \langle t, s\cdot t\rangle\in\mathcal{M}_s, a\in(s\cdot t+s), b = c+s, \langle t, c\rangle\in\mathcal{M}_s \Rightarrow a\in b \\
\hline\hline
uni[s,t,\mathcal{M}], \langle t, s\cdot t\rangle\in\mathcal{M}_s, a\in(s\cdot t+s), \langle t', b\rangle\in\mathcal{M}_s \Rightarrow a\in b \\
\hline\hline
uni[s,t,\mathcal{M}], \langle t, s\cdot t\rangle\in\mathcal{M}_s, a\in(s\cdot t+s) \Rightarrow \langle t', b\rangle\in\mathcal{M}_s \rightarrow a\in b \\
\hline
uni[s,t,\mathcal{M}], \langle t, s\cdot t\rangle\in\mathcal{M}_s, a\in(s\cdot t+s) \Rightarrow \bigwedge y(\langle t', y\rangle\in\mathcal{M}_s \rightarrow a\in y) \\
\hline
uni[s,t,\mathcal{M}], \langle t, s\cdot t\rangle\in\mathcal{M}_s, a\in(s\cdot t+s) \Rightarrow a\in(s\cdot t')
\end{array}
\;.
$$

Re 8.10iii. This is a straightforward consequence of 8.10i and 8.10ii.
Re 8.10iv. As for 6.10iv; left to the reader. QED

PROPOSITION 8.11. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(8.11i) $\Rightarrow uni[0, \mathcal{M}_a]\,;$

(8.11ii) $uni[t, \mathcal{M}_s] \Rightarrow uni[t', \mathcal{M}_s]\,.$

Proof. *Re* 8.11i. Almost trivial; left to the reader.
Re 8.11ii. Employ 8.3i and 6.7ii with two inferences according to 8.5:

$$
\begin{array}{c}
\dfrac{bas_{\mathcal{M}_s}[t', a] \Rightarrow \qquad \begin{array}{c}
c_1 = c_2 \Rightarrow c_1 + s = c_2 + s \\
\hline
uni[t, \mathcal{M}_s], \langle t, c_1\rangle\in\mathcal{M}_2, \langle t, c_2\rangle\in\mathcal{M}_2 \Rightarrow c_1 + s = c_2 + s \\
\hline\hline
uni[t, \mathcal{M}_s], c_1 + s = a, \langle t, c_1\rangle\in\mathcal{M}_2, c_2 + s = b, \langle t, c_2\rangle\in\mathcal{M}_2 \Rightarrow a = b \\
\hline\hline
uni[t, \mathcal{M}_s], stp_{\mathcal{M}_s}[t', a], stp_{\mathcal{M}_s}[t', a] \Rightarrow a = b
\end{array}}{uni[t, \mathcal{M}_s], bas_{\mathcal{M}_s}[t', a] \vee stp_{\mathcal{M}_s}[t', a], bas_{\mathcal{M}_s}[t', a] \vee stp_{\mathcal{M}_s}[t', a] \Rightarrow a = b} \\
\hline\hline
uni[t, \mathcal{M}_s], \langle t', a\rangle\in\mathcal{M}_s, \langle t', b\rangle\in\mathcal{M}_s \Rightarrow a = b \\
\hline
uni[t, \mathcal{M}_s] \Rightarrow uni[t', \mathcal{M}_s]
\end{array}
\;.
$$

QED

CONVENTIONS 8.12.
(1) $\mathfrak{C}_{\mathcal{A}_b} :\equiv uni[*_1, \mathcal{A}_b] \,\square\, \langle *_1, b + *_1\rangle\in\mathcal{A}_b \,\square\, (b + *_1)\in\mathbf{N}\,.$
(2) $\mathfrak{C}_{\mathcal{M}_a} :\equiv a\in\mathbf{N}^\circ \,\square\, uni[*_1, \mathcal{M}_a] \,\square\, \langle *_1, a\cdot *_1\rangle\in\mathcal{M}_a \,\square\, (b\cdot *_1)\in\mathbf{N}\,.$

PROPOSITION 8.13. *The following is* $\mathbf{L}^{\mathbf{i}}\mathbf{D}_\lambda$*-deducible:*

(8.13i) $\quad uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, \mathfrak{C}_{\mathcal{A}_b}, s\cdot t = b \Rightarrow (s\cdot t')\in\mathbf{N}\,;$

(8.13ii) $\quad uni[b,\mathcal{M}_a], \lbag b, \mathcal{M}_a[\![b]\!]\rbag \in \mathcal{M}_a, (a\cdot b)\in\mathbf{N}, s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N}\,.$

Proof. Re 8.13i. Employ 8.10i:

$$
\begin{array}{c}
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s \Rightarrow s\cdot t' = s\cdot t + s \\
\hline
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, s\cdot t = b \Rightarrow s\cdot t' = b + s \\
\hline
c\in\mathbf{N}^\circ \Rightarrow c\in\mathbf{N}^\circ \qquad uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, b+s=c, s\cdot t = b \Rightarrow s\cdot t' = c \\
\hline
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, c\in\mathbf{N}^\circ, b+s=c, s\cdot t = b \Rightarrow c\in\mathbf{N}^\circ \,\Box\, s\cdot t' = c \\
\hline
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, c\in\mathbf{N}^\circ, b+s=c, s\cdot t = b \Rightarrow (s\cdot t')\in\mathbf{N} \\
\hline
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, c\in\mathbf{N}^\circ \,\Box\, b+s=c, s\cdot t = b \Rightarrow (s\cdot t')\in\mathbf{N} \\
\hline
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, \bigvee y\, (y\in\mathbf{N}^\circ \,\Box\, b+s=y), s\cdot t = b \Rightarrow (s\cdot t')\in\mathbf{N} \\
\hline
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, (b+s)\in\mathbf{N}, s\cdot t = b \Rightarrow (s\cdot t')\in\mathbf{N} \\
\hline\hline
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, uni[s,\mathcal{A}_b], \lbag s, \mathcal{A}_b[\![s]\!]\rbag \in \mathcal{A}_b, (b+s)\in\mathbf{N}, s\cdot t = b \Rightarrow (s\cdot t')\in\mathbf{N} \\
\hline\hline
uni[t,\mathcal{M}_s], \lbag t, s\cdot t\rbag \in \mathcal{M}_s, \mathfrak{C}_{\mathcal{A}_b}[s], s\cdot t = b \Rightarrow (s\cdot t')\in\mathbf{N}
\end{array}\,.
$$

Re 8.13ii. Employ 8.7i and 8.7ii:

$$
\begin{array}{c}
s=a, t=b \Rightarrow (s\cdot t) = (a\cdot b) \\
\hline
c\in\mathbf{N}^\circ \Rightarrow c\in\mathbf{N}^\circ \qquad (a\cdot b) = c, s=a, t=b \Rightarrow (s\cdot t) = c \\
\hline
c\in\mathbf{N}^\circ, (a\cdot b) = c, s=a, t=b \Rightarrow c\in\mathbf{N}^\circ \,\Box\, (s\cdot t) = c \\
\hline
c\in\mathbf{N}^\circ, (a\cdot b) = c, s=a, t=b \Rightarrow \bigvee y (y\in\mathbf{N}^\circ \,\Box\, (s\cdot t) = y) \\
\hline
c\in\mathbf{N}^\circ, (a\cdot b) = c, s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N} \\
\hline
c\in\mathbf{N}^\circ \,\Box\, (a\cdot b) = c, s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N} \\
\hline
\bigvee y (y\in\mathbf{N}^\circ \,\Box\, (a\cdot b) = y), s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N} \\
\hline
(a\cdot b)\in\mathbf{N}, s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N} \\
\hline\hline
a\in\mathbf{N}^\circ, uni[b,\mathcal{M}_a], \lbag b, \mathcal{M}_a[\![b]\!]\rbag \in \mathcal{M}_a, (a\cdot b)\in\mathbf{N}, s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N} \\
\hline\hline
\mathfrak{C}_{\mathcal{M}_a}, s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N}
\end{array}\,.
$$

QED

Proposition 8.14. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(8.14i) $\Rightarrow (s \cdot 0) \in \mathbf{N}$;

(8.14ii) $a \in \mathbf{N}^\circ \Rightarrow a \in \mathbf{N}^\circ \sqcap uni[0, \mathcal{M}_a] \sqcap \langle 0, a \cdot 0 \rangle \in \mathcal{M}_a \sqcap (a \cdot 0) \in \mathbf{N}$.

Proof. *Re* 8.14ii. Employ 4.6i and 8.9i:

$$\dfrac{\dfrac{\dfrac{\Rightarrow 0 \in \mathbf{N}^\circ \qquad \Rightarrow a \cdot 0 = 0}{\Rightarrow 0 \in \mathbf{N}^\circ \sqcap (a \cdot 0) = 0}}{\Rightarrow \bigvee y (y \in \mathbf{N}^\circ \sqcap (a \cdot 0) = y)}}{\Rightarrow (a \cdot 0) \in \mathbf{N}}\ .$$

Re 8.14ii. This is a simple conjunction of 8.11i, 8.9ii and 8.14i. QED

Proposition 8.15. *The following is* $\mathbf{L^iD}_\lambda^{\mathsf{Z}\restriction 9}$*-deducible:*

(8.15i) $s \in \mathbf{N}^\circ, uni[t, \mathcal{M}_s], \langle t, s \cdot t \rangle \in \mathcal{M}_s, (s \cdot t) \in \mathbf{N} \Rightarrow (s \cdot t') \in \mathbf{N}$;

(8.15ii) $\mathsf{3}[s \in \mathbf{N}^\circ \sqcap uni[t, \mathcal{M}_s] \sqcap \langle t, s \cdot t \rangle \in \mathcal{M}_s \sqcap (s \cdot t) \in \mathbf{N}] \Rightarrow \mathfrak{C}_{\mathcal{M}_s}[t']$.

Proof. *Re* 8.15i. In the first line let $\mathcal{F}_1 :\equiv uni[t, \mathcal{M}_s]$ and $\mathcal{F}_2 :\equiv \langle t, s \cdot t \rangle \in \mathcal{M}_s$. Employ 7.3i, 7.3ii, and 8.13i:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{b \in \mathbf{N}^\circ \Rightarrow \mathfrak{C}_{\mathcal{A}_b}[0] \quad \mathsf{3}[\mathfrak{C}_{\mathcal{A}_b}[c]] \Rightarrow \mathfrak{C}_{\mathcal{A}_b}[c'] \quad \mathfrak{C}_{\mathcal{A}_b}[s], \mathcal{F}_1, \mathcal{F}_2, s \cdot t = b \Rightarrow (s \cdot t') \in \mathbf{N}}{s \in \mathbf{N}^\circ, b \in \mathbf{N}^\circ, uni[t, \mathcal{M}_s], \langle t, s \cdot t \rangle \in \mathcal{M}_s, s \cdot t = b \Rightarrow (s \cdot t') \in \mathbf{N}}{+9}}{s \in \mathbf{N}^\circ, uni[t, \mathcal{M}_s], \langle t, s \cdot t \rangle \in \mathcal{M}_s, b \in \mathbf{N}^\circ, s \cdot t = b \Rightarrow (s \cdot t') \in \mathbf{N}}}{s \in \mathbf{N}^\circ, uni[t, \mathcal{M}_s], \langle t, s \cdot t \rangle \in \mathcal{M}_s, b \in \mathbf{N}^\circ \sqcap s \cdot t = b \Rightarrow (s \cdot t') \in \mathbf{N}}}{s \in \mathbf{N}^\circ, uni[t, \mathcal{M}_s], \langle t, s \cdot t \rangle \in \mathcal{M}_s, \bigvee y (y \in \mathbf{N}^\circ \sqcap s \cdot t = y) \Rightarrow (s \cdot t') \in \mathbf{N}}}{s \in \mathbf{N}^\circ, uni[t, \mathcal{M}_s], \langle t, s \cdot t \rangle \in \mathcal{M}_s, (s \cdot t) \in \mathbf{N} \Rightarrow (s \cdot t') \in \mathbf{N}}\ .$$

Re 8.15ii. Essentially a conjunction of 8.11ii, 8.10iv, and 8.15i; left to the reader. QED

Proposition 8.16. $\mathbf{L^iD}_\lambda^{\mathsf{Z}\restriction 18} \vdash\ s \in \mathbf{N}, t \in \mathbf{N} \Rightarrow (s \cdot t) \in \mathbf{N}$.

Proof. Employ 8.14ii, 8.15ii, and 8.13ii with an inference according to 4.7viii:

$$\dfrac{\dfrac{\dfrac{a\in\mathbf{N}^\circ \Rightarrow \mathfrak{C}_{\mathcal{M}_a}[0] \qquad \begin{matrix}\mathbf{L^iD}_\lambda^{Z\restriction 9}\\ \vdots \\ 3[\mathfrak{C}_{\mathcal{M}_a}[c]] \Rightarrow \mathfrak{C}_{\mathcal{M}_a}[c']\end{matrix} \qquad \mathfrak{C}_{\mathcal{M}_a}[b], s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N}}{b\in\mathbf{N}^\circ, a\in\mathbf{N}^\circ, s=a, t=b \Rightarrow (s\cdot t)\in\mathbf{N}}\,{+9}}{a\in\mathbf{N}^\circ \mathbin{\square} s=a, b\in\mathbf{N}^\circ \mathbin{\square} t=b \Rightarrow (s\cdot t)\in\mathbf{N}}}{\bigvee y(y\in\mathbf{N}^\circ \mathbin{\square} s=y), \bigvee y(y\in\mathbf{N}^\circ \mathbin{\square} t=y) \Rightarrow (s\cdot t)\in\mathbf{N}}\,.$$

QED

REMARK 8.17. While in the case of addition, $s\in\mathbf{N}^\circ \Rightarrow s'\in\mathbf{N}^\circ$ was sufficient for proving the totality (cf. 7.2ii above), a proof of the totality of multiplication also requires $s\in\mathbf{N}, t\in\mathbf{N} \Rightarrow (s+t)\in\mathbf{N}$, *i.e.*, the totality of addition. This is what makes the number of **Z**-inferences go up.

9. Exponentiation

Given the treatment of addition and multiplication, I can dispose fairly quickly of exponentiation. Many of the following propositions will only be listed without proof.

PROPOSITION 9.1. *There exists a term $\mathcal{E}$ satisfying:*

$$\mathcal{E} = \lambda x_1 x_2 x_3 ((x_2 = 0 \wedge x_3 = 1) \vee \\ \bigvee y_1 \bigvee y_2 \bigvee y_3 (x_1 = y_1 \mathbin{\square} x_2 = y_2' \mathbin{\square} x_3 = y_3 \cdot y_1 \mathbin{\square} \langle\langle y_1, y_2\rangle, y_3\rangle \in \mathcal{E})\,.$$

Proof. As usual, this is an immediate consequence of the fixed-point property. QED

CONVENTIONS 9.2. (1) The following abbreviation is introduced to simplify presentation:

$$\mathcal{E}_s :\equiv \lambda x_2 x_3 ((x_2 = 0 \wedge x_3 = 1) \vee \\ \bigvee y_1 \bigvee y_2 \bigvee y_3 (y_1 = s \mathbin{\square} y_2' = x_2 \mathbin{\square} y_3 \cdot y_1 = x_3 \mathbin{\square} \langle\langle y_1, y_2\rangle, y_3\rangle \in \mathcal{E})\,.$$

(2) In order to save space in presentations, I shall occasionally use the following abbreviations:

$bas_{\mathcal{E}_s}[t,r]$ for $t = 0 \wedge r = 1$, and

$stp_{\mathcal{E}_s}[t,r]$ for

$$\bigvee y_1 \bigvee y_2 \bigvee y_3 (y_1 = s \,\square\, y_2' = x_2 \,\square\, y_3 \cdot y_1 = x_3 \,\square\, \langle\!\langle\langle y_1, y_2\rangle\!\rangle, y_3\rangle\!\rangle \in \mathcal{E}) \,.$$

Proposition 9.3. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(9.3i) $bas_{\mathcal{E}_s}[s', r] \Rightarrow$;

(9.3ii) $stp_{\mathcal{E}_s}[0, r] \Rightarrow$.

Definition 9.4. $s^t :\equiv \mathcal{E}_s[\![t]\!]$. I shall use $\mathcal{E}_s[\![t]\!]$ and s^t interchangeably.

Proposition 9.5. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(9.5i) $s = t \Rightarrow s^r = t^r$;

(9.5ii) $s = t \Rightarrow r^s = r^t$.

Proposition 9.6. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(9.6i) $\Rightarrow \langle\!\langle 0, 1\rangle\!\rangle \in \mathcal{E}_s$;

(9.6ii) $\langle\!\langle t, r\rangle\!\rangle \in \mathcal{E}_s \Rightarrow \langle\!\langle t', r \cdot s\rangle\!\rangle \in \mathcal{E}_s$.

Proposition 9.7. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(9.7i) $\Rightarrow s^0 = 1$;

(9.7ii) $\Rightarrow \langle\!\langle 0, s^0\rangle\!\rangle \in \mathcal{E}_s$.

Proposition 9.8. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(9.8i) $uni[s, t, \mathcal{E}], \langle\!\langle t, s^t\rangle\!\rangle \in \mathcal{E}_s, a \in (s^{t'}) \Rightarrow a \in (s^t \cdot s)$;

(9.8ii) $uni[s, t, \mathcal{E}], \langle\!\langle t, s^t\rangle\!\rangle \in \mathcal{E}_s, a \in (s^t \cdot s) \Rightarrow a \in (s^{t'})$;

(9.8iii) $uni[t, \mathcal{E}_s], \langle\!\langle t, s^t\rangle\!\rangle \in \mathcal{E}_s \Rightarrow s^{t'} = s^t \cdot s$;

(9.8iv) $uni[t, \mathcal{E}_s], 2[\langle\!\langle t, s^t\rangle\!\rangle \in \mathcal{E}_s] \Rightarrow \langle\!\langle t', s^{t'}\rangle\!\rangle \in \mathcal{E}_s$.

Proof. As usual. I only treat 9.8ii.

$$
\begin{array}{c}
\begin{array}{cc}
 & a\in(c\cdot s)\Rightarrow a\in(c\cdot s) \\ \cline{2-2}
\langle t,s^t\rangle\in\mathcal{E}_s,\langle t,b\rangle\in\mathcal{E}_s\Rightarrow\langle t,s^t\rangle\in\mathcal{E}_s\,\square\,\langle t,c\rangle\in\mathcal{E}_s & s^t=c,a\in(s^t\cdot s)\Rightarrow a\in(c\cdot s)
\end{array} \\ \hline
\langle t,s^t\rangle\in\mathcal{E}_s\,\square\,\langle t,c\rangle\in\mathcal{E}_s\rightarrow s^t=c,\langle t,s^t\rangle\in\mathcal{E}_s,a\in(s^t\cdot s),\langle t,c\rangle\in\mathcal{E}_s\Rightarrow a\in(c\cdot s) \\ \hline
uni[s,t,\mathcal{E}],\langle t,s^t\rangle\in\mathcal{E}_s,a\in(s^t\cdot s),\langle t,c\rangle\in\mathcal{E}_s\Rightarrow a\in(c\cdot s) \\ \hline
uni[s,t,\mathcal{E}],\langle t,s^t\rangle\in\mathcal{E}_s,a\in(s^t\cdot s),b=c\cdot s,\langle t,c\rangle\in\mathcal{E}_s\Rightarrow a\in b \\ \hline\hline
uni[s,t,\mathcal{E}],\langle t,s^t\rangle\in\mathcal{E}_s,a\in(s^t\cdot s),\langle t',b\rangle\in\mathcal{E}_s\Rightarrow a\in b \\ \hline\hline
uni[s,t,\mathcal{E}],\langle t,s^t\rangle\in\mathcal{E}_s,a\in(s^t\cdot s)\Rightarrow\langle t',b\rangle\in\mathcal{E}_s\rightarrow a\in b \\ \hline
uni[s,t,\mathcal{E}],\langle t,s^t\rangle\in\mathcal{E}_s,a\in(s^t\cdot s)\Rightarrow\bigwedge y(\langle t',y\rangle\in\mathcal{E}_s\rightarrow a\in y) \\ \hline
uni[s,t,\mathcal{E}],\langle t,s^t\rangle\in\mathcal{E}_s,a\in(s^t\cdot s)\Rightarrow a\in(s^{t'})
\end{array}\ .
$$

QED

Proposition 9.9. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(9.9i) $\Rightarrow uni[0,\mathcal{E}_s]$;

(9.9ii) $uni[t,\mathcal{E}_s]\Rightarrow uni[t',\mathcal{E}_s]$.

Convention 9.10. $\mathfrak{C}_{\mathcal{E}_a} :\equiv a\in\mathbf{N}^\circ\,\square\,uni[*_1,\mathcal{E}_a]\,\square\,\langle *_1,a^{*_1}\rangle\in\mathcal{E}_a\,\square\,a^{*_1}\in\mathbf{N}$.

Proposition 9.11. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(9.11i) $uni[t,\mathcal{E}_s],\langle t,s^t\rangle\in\mathcal{E}_s,\mathfrak{C}_{\mathcal{M}_b}[s],s^t=b\Rightarrow s^{t'}\in\mathbf{N}$;

(9.11ii) $a\in\mathbf{N}^\circ\,\square\,uni[b,\mathcal{E}_a]\,\square\,\langle x,a^b\rangle\in\mathcal{E}_a\,\square\,a^b\in\mathbf{N},s=a,t=b\Rightarrow s^t\in\mathbf{N}$.

Proposition 9.12. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(9.12i) $\Rightarrow a^0\in\mathbf{N}$;

(9.12ii) $a\in\mathbf{N}^\circ\Rightarrow a\in\mathbf{N}^\circ\,\square\,uni[0,\mathcal{E}_a]\,\square\,\langle 0,a^0\rangle\in\mathcal{E}_a\,\square\,a^0\in\mathbf{N}$.

Proof. *Re* 9.12i. Employ 9.7i:

$$
\begin{array}{c}
\Rightarrow 1\in\mathbf{N}^\circ \qquad \Rightarrow a^0=1 \\ \hline
\Rightarrow 1\in\mathbf{N}^\circ\,\square\,a^0=1 \\ \hline
\Rightarrow\bigvee y(y\in\mathbf{N}^\circ\,\square\,a^0=y) \\ \hline
\Rightarrow a^0\in\mathbf{N}
\end{array}\ .
$$

QED

Proposition 9.13. *The following is* $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 18}$*-deducible:*

(9.13i) $s\in\mathbf{N}^\circ, uni[t,\mathcal{E}_s], \langle t, s^t\rangle\in\mathcal{E}_s, s^t\in\mathbf{N} \Rightarrow s^{t'}\in\mathbf{N}\,;$

(9.13ii) $3\,[s\in\mathbf{N}^\circ \,\square\, uni[t,\mathcal{E}_s] \,\square\, \langle t, s^t\rangle\in\mathcal{E}_s \,\square\, s^t\in\mathbf{N}]$

$$\Rightarrow s\in\mathbf{N}^\circ \,\square\, uni[t',\mathcal{E}_s] \,\square\, \langle t', s^{t'}\rangle\in\mathcal{E}_s \,\square\, s^{t'}\in\mathbf{N}\,.$$

Proof. Re 9.13i. Employ 8.14ii, 8.15ii, and 9.11i with an inference according to 4.7ii. In the first line, let $\mathfrak{E}[s,t]$ stand for $uni[t,\mathcal{E}_s], \langle t, s^t\rangle\in\mathcal{E}_s$:

$$\dfrac{\dfrac{\dfrac{\dfrac{b\in\mathbf{N}^\circ \Rightarrow \mathfrak{C}_{\mathcal{M}_b}[0] \qquad 3\,[\mathfrak{C}_{\mathcal{M}_b}[c]] \Rightarrow \mathfrak{C}_{\mathcal{M}_b}[c'] \qquad \overset{\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 9}}{\vdots}\; \mathfrak{E}[s,t], \mathfrak{C}_{\mathcal{M}_b}[s], s^t=b \Rightarrow s^{t'}\in\mathbf{N}}{s\in\mathbf{N}^\circ, uni[t,\mathcal{E}_s], \langle t, s^t\rangle\in\mathcal{E}_s, b\in\mathbf{N}^\circ, s^t=b \Rightarrow s^{t'}\in\mathbf{N}}\,{+9}}{s\in\mathbf{N}^\circ, uni[t,\mathcal{E}_s], \langle t, s^t\rangle\in\mathcal{E}_s, b\in\mathbf{N}^\circ \,\square\, s^t=b \Rightarrow s^{t'}\in\mathbf{N}}}{s\in\mathbf{N}^\circ, uni[t,\mathcal{E}_s], \langle t, s^t\rangle\in\mathcal{E}_s, \bigvee y\,(y\in\mathbf{N}^\circ \,\square\, s^t=y) \Rightarrow s^{t'}\in\mathbf{N}}}{s\in\mathbf{N}^\circ, uni[t,\mathcal{E}_s], \langle t, s^t\rangle\in\mathcal{E}_s, s^t\in\mathbf{N} \Rightarrow s^{t'}\in\mathbf{N}}\;.$$

Re 9.13ii.

$$\dfrac{\dfrac{\dfrac{\dfrac{\overset{\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 18}}{\vdots}\; s\in\mathbf{N}^\circ, uni[t,\mathcal{E}_s], \langle t, s^t\rangle\in\mathcal{E}_s, s^t\in\mathbf{N} \Rightarrow s^{t'}\in\mathbf{N}}{s\in\mathbf{N}^\circ, 2\,[uni[t,\mathcal{E}_s]], 2\,[\langle t, s^t\rangle\in\mathcal{E}_s], s^t\in\mathbf{N} \Rightarrow \langle t', s^{t'}\rangle\in\mathcal{E}_s \,\square\, s^{t'}\in\mathbf{N}}}{s\in\mathbf{N}^\circ, 3\,[uni[t,\mathcal{E}_s]], 2\,[\langle t, s^t\rangle\in\mathcal{E}_s], s^t\in\mathbf{N} \Rightarrow uni[t,\mathcal{E}_s] \,\square\, \langle t', s^{t'}\rangle\in\mathcal{E}_s \,\square\, s^{t'}\in\mathbf{N}}}{s\in\mathbf{N}^\circ, 3\,[uni[t,\mathcal{E}_s]], 3\,[\langle t, s^t\rangle\in\mathcal{E}_s], 3\,[s^t\in\mathbf{N}] \Rightarrow uni[t,\mathcal{E}_s] \,\square\, \langle t', s^{t'}\rangle\in\mathcal{E}_s \,\square\, s^{t'}\in\mathbf{N}}}{3\,[s\in\mathbf{N}^\circ \,\square\, uni[t,\mathcal{E}_s] \,\square\, \langle t, s^t\rangle\in\mathcal{E}_s \,\square\, s^t\in\mathbf{N}] \Rightarrow s\in\mathbf{N}^\circ \,\square\, uni[t,\mathcal{E}_s] \,\square\, \langle t', s^{t'}\rangle\in\mathcal{E}_s \,\square\, s^{t'}\in\mathbf{N}}\;.$$

QED

Proposition 9.14. $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 27} \vdash\ s\in\mathbf{N}, t\in\mathbf{N} \Rightarrow s^t\in\mathbf{N}\,.$

Proof. Employ 9.12ii, 9.13ii, and 9.11ii with an inference according to 4.7viii:

$$\dfrac{\dfrac{\dfrac{\dfrac{a\in\mathbf{N}^\circ \Rightarrow \mathfrak{C}_{\mathcal{E}_a}[0] \qquad \overset{\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 18}}{\vdots}\; 3[\mathfrak{C}_{\mathcal{E}_a}[c]] \Rightarrow \mathfrak{C}_{\mathcal{E}_a}[c'] \qquad \mathfrak{C}_{\mathcal{E}_a}[b], s=a, t=b \Rightarrow (s^t)\in\mathbf{N}}{a\in\mathbf{N}^\circ, s=a, b\in\mathbf{N}^\circ, t=b \Rightarrow s^t\in\mathbf{N}}\,{+9}}{a\in\mathbf{N}^\circ \,\Box\, s=a, b\in\mathbf{N}^\circ \,\Box\, t=b \Rightarrow s^t\in\mathbf{N}}}{\bigvee y(y\in\mathbf{N}^\circ \,\Box\, s=y), \bigvee y(y\in\mathbf{N}^\circ \,\Box\, t=y) \Rightarrow s^t\in\mathbf{N}}}{s\in\mathbf{N}, t\in\mathbf{N} \Rightarrow s^t\in\mathbf{N}}\ .$$

QED

REMARK 9.15. The totality of exponentiation presupposes that of addition and multiplication: that adds up to 27 **Z**-inferences.

10. The series of primitive recursive functions continued

So far I have considered the following two-place primitive recursive functions:

$$\phi_0(a,b) = a+b\,,$$
$$\phi_1(a,b) = a\cdot b\,,$$
$$\phi_2(a,b) = a^b\,.$$

They can be seen as forming the beginning part of a series:[24] just as multiplication is the iteration of addition in the form

$$a+a+\ldots+a = a\cdot \mathrm{n}\,,$$

exponentiation is the iteration of multiplication in the form

$$a\cdot a\cdot\ldots\cdot a = a^{\mathrm{n}}\,.$$

This can be continued to a super-exponentiation:

$$a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}$$

[24] As has been done in [12], p. 185 ([27], p. 388), and also [13], p. 336, to motivate the formulation of the Ackermann function.

which would be governed by the following recursion equations:

$$\phi_3(a,0) = a\,,$$
$$\phi_3(a,b') = a^{\phi_3(a,b)}\,, \quad \text{or:} \quad \phi_2(a,\phi_3(a,b))\,.$$

In general, a series of functions ϕ_{n} can be defined for $\mathrm{n} > 2$ as follows:

$$\phi_{\mathrm{n}'}(a,0) = a\,,$$
$$\phi_{\mathrm{n}'}(a,b') = \phi_{\mathrm{n}}(a,\phi_{\mathrm{n}'}(a,b))\,.$$

In terms of their recursion equations, we have the following:

$$\phi_0(a,b) = \begin{cases} a\,, & \text{if } b = 0\,; \\ \phi_0(a,c)'\,, & \text{if } b = c' \text{ for some } c\,. \end{cases}$$

$$\phi_1(a,b) = \begin{cases} 0\,, & \text{if } b = 0\,; \\ \phi_0(\phi_1(a,c),a)\,, & \text{if } b = c' \text{ for some } c\,. \end{cases}$$

$$\phi_2(a,b) = \begin{cases} 1\,, & \text{if } b = 0\,; \\ \phi_1(\phi_2(a,c),a)\,, & \text{if } b = c' \text{ for some } c\,. \end{cases}$$

$$\phi_3(a,b) = \begin{cases} a\,, & \text{if } b = 0\,; \\ \phi_2(\phi_3(a,c),a)\,, & \text{if } b = c' \text{ for some } c\,. \end{cases}$$

$$\vdots$$

$$\phi_{\mathrm{n}'}(a,b) = \begin{cases} a\,, & \text{if } b = 0\,; \\ \phi_{\mathrm{n}}(\phi_{\mathrm{n}'}(a,c),a)\,, & \text{if } b = c' \text{ for some } c\,. \end{cases}$$

The recursion equations for the functions ϕ_{n} are $\mathbf{L^{i}D}_{\lambda}^{\mathrm{Z}\restriction 4}$-deducible, and that for all $\mathrm{n} \in \mathbb{N}$. It is the proof of the totality of ϕ_{n} that requires recourse to the totality of the functions ϕ_{k} with $\mathrm{k} < \mathrm{n}$ and thus can be expected to be $\mathbf{L^{i}D}_{\lambda}^{\mathrm{Z}\restriction 9\mathrm{n}+9}$-deducible.

Now it is well-known that Ackermann's function can be presented as a kind of totalization over this series, by turning the index number of the ϕ_{k} into an additional argument. Its recursion equations are no longer $\mathbf{L^{i}D}_{\lambda}^{\mathrm{Z}}$-deducible; we need something like the reinforced necessity operator that I introduced in [23], pp. 136–159. But this will be the topic of a follow-up to the present paper in which I will consider the complexity of k-recursive functions in more detail.

11. Z-inferences as a measure of complexity[25]

Towards the end of his famous address entitled *Über das Unendliche* ("On the infinite"), Hilbert declared (in translation):[26]

> The role that remains to the infinite is ⟦...⟧ merely that of an idea—if, in accordance with Kant's words, we understand by an idea a concept of reason that transcends all experience and through which the concrete is completed so as to form a totality⟦.⟧[27]

Hilbert's proof theory was meant to justify the use of classical logic for this supposed role of infinity as "merely that of an idea", but, cautiously put, his program was not successful. If this is taken to indicate that the role of "a concept of reason that transcends all experience" cannot simply be reduced to that of a neutral supplementation, then the question regarding the nature of the infinite and its appropriate logic would have to be raised again.

Intuitionistic logic, despite its declared aim to overcome classical logic in its treatment of the infinite is not a suitable alternative: it remains within a somewhat classical paradigm. As Girard put it:

> Classical and intuitionistic logics deal with *stable* truths:
>
> If A and A ⇒ B , then B, *but A still holds.*[28]

This is a hallmark of contraction and that's why abandoning contraction recommends itself when confronting the possibility of unstable truths — something that may well happen when dealing with infinity.

With contraction available, resources can be multiplied *ad libitum* at no extra costs.[29] This proves vital when it comes to formulating a term that is to capture *exactly* the natural numbers.[30] The induction

[25] The idea put forward in this section is very tentative, indeed, and should be taken with a pound of salt. In any case, it is what motivated my investigations into how many **Z**-inferences are needed to prove the totality of certain primitive recursive functions.

[26] [27], p. 367.

[27] [27], p. 392.

[28] [9], p. 1.

[29] In Girard's diction "*contraction* is the fingernail of infinity" ([7], p. 78).

[30] It is easy enough to provide a term that captures all natural numbers, but the point for induction is that it is *only* the natural numbers that are captured. This is what I labeled "exclusion principle" in remarks 116.6 and 119.1 in [21], for example.

step is available as often as one likes, but it only has to be accounted for once. Without contraction this changes: with assumptions having to be accounted for, the formulation of the induction step requires special attention to the effect of specifying how often it is available. This is what **Z**-inferences were designed to accomplish.[31] They provide an alternative approach to infinity — and this approach is what I want to propose as a basis for a measure of complexity: how many **Z**-inferences are required in the proof of a particular result.

In view of these considerations I should emphasize that my approach is not so much aimed at a notion of *computational* complexity, but more at something like a *metaphysical* complexity.

Now recall the results of the foregoing sections:[32]

1. The totality of the predecessor function is $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 1}$-deducible;
2. the recursion equations of primitive recursive functions are $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 4}$-deducible;
3. the totality of addition is $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 9}$-deducible;
4. the totality of multiplication is $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 18}$-deducible;
5. the totality of exponentiation is $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 27}$-deducible;
6. In general, the totality of φ_{n} can be expected to be $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction 9\mathrm{n}+9}$-deducible.

How can this be linked to a notion of complexity? Not surprisingly, perhaps, the suggestion I want to make evokes consistency proofs. The idea is that the number of **Z**-inferences in deductions determines how high an induction is needed for a consistency proof. As is well-known, for the system which allows no **Z**-inference at all ($\mathbf{L^iD}_\lambda$), an induction up to ω suffices, but for the realm beyond that I need a conjecture.

CONJECTURE 11.1. *The consistency of* $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction \mathrm{n}}$ *can be established by an induction up to* $\omega^{\mathrm{n}+1}$.

Comment. This would be in accordance with the consistency of $\mathbf{L^iD}_\lambda^{\mathrm{Z}}$ being ω^ω-provable: every $\mathbf{L^iD}_\lambda^{\mathrm{Z}}$-deduction is a $\mathbf{L^iD}_\lambda^{\mathrm{Z}\restriction \mathrm{n}}$ for some $\mathrm{n} \in \mathbb{N}$.

[31] There are alternative ways of doing this such as [16] and [19] neither of which, however, is designed to capture full primitive recursion.

[32] It must be understood that the following deducibility claims indicate upper bounds only, *i.e.*, I haven't established that any of the proofs, be that of the recursion equations or the totality of addition, multiplication, or exponentiation cannot be reduced to less **Z**-inferences.

In view of the foregoing conjecture, the following hierarchy is suggested by taking the proof of the totality of a function as the basis for a measure of complexity:

1. the predecessor function is assigned the complexity ω^2 ;
2. addition is of a complexity $\geq \omega^2$ and $\leq \omega^{10}$;
3. multiplication is of a complexity $\geq \omega^3$ and $\leq \omega^{19}$;
4. exponentiation is of a complexity $\geq \omega^4$ and $\leq \omega^{28}$;
5. in general, the function φ_{n} is of a complexity $\geq \omega^{\mathrm{n}+1}$ and $\leq \omega^{9\mathrm{n}+10}$.

If this is continued as suggested at the end of the last section, the complexity of a function defined by nested double recursion can be expected to be somewhere above ω^ω. Thus the measure of complexity suggested here differs quite significantly from the one suggested by Rózsa Péter's work according to which nested n-fold recursion would have a complexity of ω^n. This difference has its origin in a different treatment of infinity as expressed in the formulation of the term $\mathbf{N}^\circ$.

What remains is the question of whether this hierarchy is immune to the possibilities of reducing an induction up to ω^2, for instance, to an ordinary one. Of course, my immediate response would be to direct attention, once again, to the different treatment of infinity. The point is simply that these reductions are based on a *classical* form of induction, *i.e.*, one involving classical logic, in particular contraction, albeit on a meta level. On the basis of the present resource conscious logic, not even course-of-value induction (or: strong induction) is reducible to ordinary induction. In the classical case (of suitable higher order), course-of-value induction can be established in the form

$$s \in \mathbf{N}, \bigwedge y (\bigwedge x (x < y \to \mathfrak{F}[x]) \to \mathfrak{F}[y] \Rightarrow \mathfrak{F}[s] \,,$$

where $\mathbf{N}$ is the term

$$\lambda x \bigwedge y (\bigwedge z (z \in y \to z' \in y) \to (0 \in y \to x \in y)) \,,$$

whereas its dialectical counterpart requires a necessity operator:[33]

$$s \in \mathbf{N}^\circ, \Box \bigwedge y (\bigwedge x (x < y \to \mathfrak{F}[x]) \to \mathfrak{F}[y] \Rightarrow \mathfrak{F}[s] \,.$$

[33] Cf. [22], p. 676. 7iii.

Differently put, if strong induction (in its classical form) is captured by the term

$$\mathbf{N}^{+} :\equiv \lambda x \bigwedge y(\bigwedge z(\bigwedge z_1(z_1 < z \to z_1 \in y) \to z \in y) \to x \in y)\,,$$

then the following is classically provable:

$$(11.48) \qquad s \in \lambda x \bigwedge y(\bigwedge z(z \in y \to z' \in y) \to (0 \in y \to x \in y)) \Rightarrow s \in \mathbf{N}^{+}\,.$$

To see this, take $\mathfrak{C} :\equiv \bigwedge y(y < *_1 \to y \in \mathbf{N}^{+})$ and confirm that the following is provable, classically as well as dialectically:

$$\Rightarrow \mathfrak{C}[0]\,,$$
$$\mathfrak{C}[a], \bigwedge x(\mathfrak{C}[x] \to x \in \mathbf{N}^{+}) \Rightarrow \mathfrak{C}[a']\,,$$
$$\mathfrak{C}[s'] \Rightarrow s \in \mathbf{N}^{+}\,.$$

In the classical case, this yields 11.48 by means of a simple induction, but not so in the dialectical case. It's the side wff $\bigwedge x(\mathfrak{C}[x] \to x \in \mathbf{N}^{+})$ which makes things more complicated. Instead of a proof of 11.48 by means of a simple induction one only gets

$$\mathbf{L^{i}D}_{\lambda}^{Z\restriction 2} \vdash \Rightarrow \mathbf{N}^{\circ} \subseteq \lambda x \bigwedge y(\Box \bigwedge z(\bigwedge z_1(z_1 < z \to z_1 \in y) \to z \in y) \to x \in y)\,,$$

i.e., the reduction becomes more costly of deductive means. This is resource consciousness manifesting itself.

This, I contend, matters in the case of induction up to ω^2 (and beyond, of course) as well. To be sure, this is not meant to serve as a *proof* of the impossibility of reducing induction up to ω^2 to ordinary induction in contraction free logic, but just to indicate, how a familiar classical strategy may turn sour in the case of contraction free logic: a reduction of induction up to ω^2 to an induction up to ω may require an induction up to ω^2 — provided, of course, one works within a contraction free logic.

12. Appendix: Natural numbers and elements of $\boldsymbol{\Psi}$

The term $\mathbf{N}^{\circ}$, which is designed to represent the set of natural numbers on the formal level, has been introduced *via* a notion of weak implication which, in turn, was based on a notion of having available a certain wff a certain *number of times*. Having available a wff a certain *number* of times, however, does not require a full-fledged notion of natural number, but only that of a certain *proto-number*, elements of the collection $\boldsymbol{\Psi}$, which was captured in the formal notion $\breve{\mathbf{\Pi}}^{\circ}$.

In definition 2.10 (2) above, a correspondence has been introduced which provided a link between natural numbers (*i.e.*, elements of $\mathbb{N}$) and elements of $\boldsymbol{\Psi}$. I shall now provide a term that will take care of this correspondence on the formal level, *i.e.*, relate $\breve{\mathbf{\Pi}}$ and $\mathbf{N}^{\circ}$. In character this term resembles a primitive recursive function, only that it doesn't have values in the natural numbers, but Ψ instead.

Proposition 12.1. *There exists a fixed point* $\tilde{\nu}$ *such that*

$$\mathbf{L^iD}_\lambda \vdash \ \tilde{\nu} = \lambda x_1 x_2 ((x_1 = 0' \sqcap x_2 = I) \vee \bigvee y_1 \bigvee y_2 (x_1 = y_1'' \sqcap x_2 = y_2^I \sqcap \langle y_1', y_2 \rangle \in \tilde{\nu}))\,.$$

The treatment is essentially the same as for addition (treated as a one-place function) only that the values are not in the natural numbers. I just list the relevant properties without proof.

Proposition 12.2. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(12.2i) $\Rightarrow \langle 0', I \rangle \in \tilde{\nu}\,;$

(12.2ii) $\langle s', t \rangle \in \tilde{\nu} \Rightarrow \langle s'', t^I \rangle \in \tilde{\nu}\,.$

Proposition 12.3. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(12.3i) $\Rightarrow \tilde{\nu}[\![0']\!] = I\,;$

(12.3ii) $\Rightarrow \langle 0', \tilde{\nu}[\![0']\!] \rangle \in \tilde{\nu}\,.$

Proposition 12.4. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(12.4i) $uni[s', \tilde{\nu}], \langle s', \tilde{\nu}[\![s']\!] \rangle \in \tilde{\nu} \Rightarrow \tilde{\nu}[\![s'']\!] = \tilde{\nu}[\![s']\!]^I\,;$

(12.4ii) $uni[s', \tilde{\nu}], \langle s', \langle s', \tilde{\nu}[\![s']\!] \rangle \in \tilde{\nu} \Rightarrow \langle s'', \tilde{\nu}[\![s'']\!] \rangle \in \tilde{\nu}\,.$

Proposition 12.5. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(12.5i) $\Rightarrow uni[0', \tilde{\nu}]\,;$

(12.5ii) $uni[s', \tilde{\nu}] \Rightarrow uni[s'', \tilde{\nu}]\,.$

Proposition 12.6. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(12.6i) $\Rightarrow uni[0', \tilde{\nu}] \sqcap \langle 0', I \rangle \in \tilde{\nu}\,;$

(12.6ii) $2\,[uni[a, \tilde{\nu}] \sqcap \langle a, \tilde{\nu}[\![a]\!] \rangle \in \tilde{\nu}] \Rightarrow uni[a', \tilde{\nu}] \sqcap \langle a', \tilde{\nu}[\![a']\!] \rangle \in \tilde{\nu}\,.$

Proposition 12.7. $\mathbf{L^iD}_\lambda^{Z\restriction 4} \vdash\ s\in\mathbf{N}^\circ \Rightarrow \tilde{\mathrm{v}}\llbracket s''\rrbracket = \tilde{\mathrm{v}}\llbracket s'\rrbracket^I$.

Proof. As usual, employ an inference according to schema 4.7vii, in the present case with 12.6i and 12.6ii. QED

Next comes totality in the sense of showing that $s'\in\mathbf{N}^\circ \Rightarrow \tilde{\mathrm{v}}\llbracket s'\rrbracket\in\breve{\mathbf{\Pi}}$.

Proposition 12.8. *The following is* $\mathbf{L^iD}_\lambda$*-deducible:*

(12.8i) $\Rightarrow uni[0',\tilde{\mathrm{v}}] \square \langle 0',\tilde{\mathrm{v}}\llbracket x\rrbracket\rangle\in\tilde{\mathrm{v}} \square \tilde{\mathrm{v}}\llbracket 0'\rrbracket\in\breve{\mathbf{\Pi}}$;

(12.8ii) $3\,[uni[a',\tilde{\mathrm{v}}] \square \langle a',\tilde{\mathrm{v}}\llbracket a'\rrbracket\rangle\in\tilde{\mathrm{v}} \square \tilde{\mathrm{v}}\llbracket a'\rrbracket\in\breve{\mathbf{\Pi}}] \Rightarrow$

$$uni[a'',\tilde{\mathrm{v}}] \square \langle a'',\tilde{\mathrm{v}}\llbracket a''\rrbracket\rangle\in\tilde{\mathrm{v}} \square \tilde{\mathrm{v}}\llbracket a''\rrbracket\in\breve{\mathbf{\Pi}} .$$

Proposition 12.9. $\mathbf{L^iD}_\lambda^{Z\restriction 9} \vdash\ s'\in\mathbf{N}^\circ \Rightarrow \tilde{\mathrm{v}}\llbracket s'\rrbracket\in\breve{\mathbf{\Pi}}$.

Proof. As usual, employ an inference according to schema 4.7viii, in the present case with 12.8i and 12.8ii. QED

Proposition 12.10. $\mathbf{L^iD}_\lambda \vdash \Rightarrow \tilde{\mathrm{v}}\llbracket 0\rrbracket = \mathcal{V}$

Proof. Straightforward; left to the reader. QED

Remarks 12.11. (1) It will be obvious that $[A/\tilde{\mathrm{v}}\llbracket n'\rrbracket] \leftrightarrow [A]^{\cdot \mathrm{n}}$ can be established by a meta-theoretical induction on n. In other words, the meta-theoretical notion of $[A]^{\cdot \mathrm{n}}$ can be replaced by the formal notion $[A/\tilde{\mathrm{v}}\llbracket n'\rrbracket]$.
(2) What 12.10 says is basically that the function $\tilde{\mathrm{v}}$ is not "defined" for the argument 0 in the sense of not having a value in $\mathbf{\Psi}$.

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The Mimetic Life of Captain Cook and Sovereignty in Australia

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ABSTRACT. This paper argues that sovereignty in Australia is mimetic. The nature of sovereignty in Australia must be understood in the colonial context. Anglo-European sovereignty produces imperfect copies of itself (native title, civilised savage, traditional laws and customs) in order to secure itself as original and authoritative as a strategy and effect of its own power. However, as part of the mimetic nature of sovereignty in Australia, Anglo-European sovereignty is always at risk of being undermined by Aboriginal claims that they too have sovereignty.

Contents

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Introduction

> It is only the continuance of exploitation and the filling of the gaps with pragmatism, while all else continues as before, that washes the shores of where Cook walked before.[2]

The subject of this paper is the figure of Captain James Cook. In particular, I am concerned with the link between Captain Cook and sovereignty in the country we now call Australia. The figure of Captain Cook, like sovereignty itself, is contested in Australia along the lines of coloniser and colonised, Anglo-European and Aboriginal. For Anglo-European Australia Captain Cook is the celebrated "discoverer" of the east coast of the continent, claiming it as a colonial possession of the British Crown and paving the way for the occupation of Australia by the British in 1788. Captain Cook is a symbol, standing as a metonym, in both law and Anglo-European history for the assertion of British sovereignty over Australia. In contrast, Captain Cook is a figure of great ambivalence in Aboriginal accounts and narratives of colonisation and is largely characterised as villainous as well as a usurper of Aboriginal sovereignty and control over land. There is little doubt that Captain Cook has also become a metonymic symbol for sovereignty in Aboriginal Australia.

In this paper I take up one of these Aboriginal accounts of Captain Cook as a platform for my discussion of sovereignty in Australia. I will look at Captain Cook through the prism of *Too Many Captain Cooks*, a Rembarrnga account of Captain Cook from Arnhem Land in the Northern Territory (NT) of Australia. The extraordinary feature of *Too Many Captain Cooks* is that Captain Cook appears as a dreaming ancestor of the Rembarrnga. From the outside looking in the Rembarrnga Captain Cook is almost unrecognisable as Captain Cook but for his name, his stated association with the material things of Anglo-Europeans and his link with, so to speak, his "namesake" — the too many Captain Cooks that eventually follow him to Australia. Despite its almost unrecognisable incorporation of Captain Cook, I contend that *Too Many Captain Cooks* is mirror-like (it reflects back to "us") and tells us something about the nature of sovereignty in Australia (Rembarrnga, Aboriginal and, importantly, the Crown's sovereignty). There is something important in the

[2] Watson I, 'Buried Alive', *Law and Critique*, 13:3 (2002), 259.

very act of the copying of Captain Cook that gives us an insight into sovereignty in Australia.

In this paper I will argue that sovereignty is mimetic — there is an intrinsic relationship between Anglo-European and Aboriginal sovereignty in Australia. The literature on sovereignty has tended to treat Aboriginal and Anglo-European sovereignty as separate and distinctive. If we take Captain Cook to stand for sovereignty, a reading of *Too Many Captain Cooks* tells us that Aboriginal sovereignty and the Crown's sovereignty are in one way or another intimately tied. Some formulations of Aboriginal sovereignty have started to look more and more like the Crown's sovereignty, once we move past the Crown as a symbol of sovereignty to its institutional manifestation in the form of the State. In addition, the way in which the Crown's sovereignty is asserted in Australia (as an indivisible sovereignty) clearly contests the existence of Aboriginal sovereignty in Australia. This paper seeks to tease out exactly how the intimate relationship between, generally speaking, these two sovereignties play out. To aid this discussion I will draw on three theories of mimesis (Michael Taussig, Homi K Bhabha and René Girard) to discuss the link between Captain Cook and Captain Cook, sovereignty and sovereignty. I will relate these theories to a reading of *Too Many Captain Cooks* and to the influential theory of the nature of western sovereign power in the work of Thomas Hobbes. The discussion will be taken to the key High Court judgment in *Mabo v Queensland (No 2)*.[3]

The purpose of this paper is to give us an insight into the nature of sovereignty in the Australian colonial setting. I will show that Anglo-European sovereignty produces impoverished copies of itself in order to secure itself as original and authoritative as a strategy and effect of its own power. However, as part of the mimetic nature of sovereignty in Australia, Anglo-European sovereignty is always at risk of being undermined by Aboriginal claims that they too have sovereignty. This raises the possibility that while the mimetic nature of sovereignty in Australia is a strategy of domination, it also contains the seeds for its own contestation and provides a platform for a more reciprocal understanding of sovereign power.

[3] [1992] HCA 23; (1992) 175 CLR 1. (Hereafter *Mabo*. Subsequent references are to the HCA report).

1. Many Captain Cooks

1.1. *Too Many Captain Cooks* — A Rembarrnga Dreaming of Captain Cook

The following discussion of the Rembarrnga dreaming is taken from the video *Too Many Captain Cooks*.[4] The video records Paddy Wainburranga painting, singing and telling the story of Captain Cook. There are two Captain Cooks in *Too Many Captain Cooks*: Captain Cook the 'law man' and ancestor of the Rembarrnga and the 'New Captain Cooks' (the British). It appears that Wainburranga is disputing that white people know the real Captain Cook, as Captain Cook 'was a law man' from millions of years ago and not the Captain Cook of 200 years ago. Wainburranga says that his generation knew Captain Cook; the geese and cockatoo (amongst a list of others) knew Captain Cook in the time that they were human. Captain Cook was like, he recounts, Adam and Eve, though Adam and Eve were 'only half way'. Captain Cook was, as Wainburranga puts it, 'there first'.

Wainburranga tells us that Captain Cook was from Mosquito Island, an island east of Papua New Guinea. Captain Cook came to Sydney Harbour (sometimes called Sydney Island by Wainburranga) in his boat with his two wives. There were, we are told, millions of people in Australia when Captain Cook came, but he did not 'interfere' with them. Captain Cook brought in his boat useful material things of the white man, including blankets, calico, trousers, axes, steel knives and, even, flags.

Wainburranga tells us that Captain Cook was working on his boat at Sydney Island. 'Satan', who lived on the other side of Sydney Island alone with no family, wanted to kill Captain Cook and take his wives. Satan asked Captain Cook if he had magic, to which Captain Cook answered, 'no'. We are told that Captain Cook says to Satan, as he has a magic bone (Captain Cook only has a stone axe) that they should fight hand to hand. Captain Cook manages to kill Satan in the fight. Wainburranga tells us that Captain Cook becomes the 'owner of the country' and is also 'the boss of Mosquito Island'. Captain Cook sails back to Mosquito Island, but on his return he is speared by his own relatives. Wounded, Captain Cook makes it back to Sydney Island but dies there. Then Wainburranga

[4] McDonald P, *Too Many Captain Cooks* (Civic Square, ACT, 1988).

says, '*other people started thinking they could make Captain Cook another way'*.[5]

At this juncture the dreaming appears to morph into something more familiar — a historical account of colonisation. We are told that new people, 'all his sons', 'New Captain Cooks' come to Australia and their families follow over. While Captain Cook never made war with Aboriginal people, the New Captain Cooks killed many Aboriginal people first in Sydney and then 'taking over'. 'From the New Captain Cooks 100 years ago, 200 years ago', Wainburranga tells us, 'too many Captain Cooks, too many Captain Cooks'. Wainburranga finishes the story by saying that 'we' know and respect only one Captain Cook and that 'no one can change our law, no one can change our culture ... we have the story of Captain Cook'.

Captain Cook is an incredibly ubiquitous figure in Aboriginal Australia. The Rembarrnga dreaming is just one Aboriginal story in which Captain Cook is the central focus and figure. Captain Cook appears in Aboriginal stories from every corner of the continent, including in stories belonging to Aboriginal communities located in areas which, according to Captain Cook's journal, Captain Cook did not set foot in nor sail near. Arnhem Land is situated in the north eastern corner of the Northern Territory and is not, at least according to Captain Cook or his botanist Joseph Banks, a place visited during the Endeavour exploration. While the Rembarrnga story of Captain Cook has all the indicia of what is called the dreaming, other stories fall somewhere between a contemporaneous oral history of first contact or subsequent contact and the dreaming. For example, in a Gurindji story (from South West of Arnhem Land) about Captain Cook and Ned Kelly, Cook/Kelly are figurative for villainous and friendly Europeans. Ned Kelly is the infamous bush ranger from the 19th century who, as far as we know, never ventured near Gurindji country. In the story, Ned Kelly was a pastoralist and friend of Aborigines, while Captain Cook looked 'at the land and saw that it was very good and wanted it for himself' and killed Ned Kelly.[6]

How are we to explain the ubiquity of Captain Cook in Aboriginal Australia with a view to developing a reading of *Too Many Captain*

[5] Emphasis added.

[6] Maddox K, 'Myth, History and a Sense of Oneself'in J Beckett *Past and Present* (Canberra, 1988), 18.

Cooks? Kenneth Maddox sees the Captain Cook stories as the emergence of political 'myths', which 'not only explain or refer to a state of affairs but envisage an alternative to it'.[7] In the Gurindji story while Captain Cook is considered to be a villain, the Ned Kelly type figure personifies the possibility of amicable relations between Aboriginal and Anglo-European.[8] The various Aboriginal accounts of Captain Cook represent, what Michael Jackson calls, a 'transmigration of a name'.[9] Jackson discusses the use of the name of Alexander the Great in different historical and cultural milieus as a political strategy to underscore political power, providing some support for Maddox's reading. The Macedonian world-conqueror has even become an ancestor of a ruling lineage in a remote West African society in contemporary times. Reflecting on the impact that the transmigration of a name has on the definitive historical figure, Jackson poses a question:

> Where then is the real Alexander, amid all these versions in which ancient events have become metamorphosed according to the preoccupations of different societies in different epochs?[10]

The answer given by C B Welles to the question posed by Jackson is: 'there have been many Alexanders. Probably there will never be a definitive Alexander'.[11]

Both Maddox and Jackson provide useful starting points by highlighting the political nature of transmigration figures but there are limitations to each analysis in this context. Maddox, for example, adheres to a strict separation between myth and history (there is a definitive or authoritative Captain Cook) and does not provide a platform to discuss the incorporation of Captain Cook into the Rembarrnga dreaming as more than metaphorical or allegorical. Jackson applies a less rigid view of myth and history. However, his analysis does not lend itself to the situation in which we find ourselves, where there is a contest over the name and representation of Captain Cook between, broadly speaking, Aboriginal and Anglo-European Australia. As we will see in the next section Captain Cook is an equally important figure for Anglo-European Australia.

[7] Ibid, 28.

[8] Ibid, 21.

[9] Jackson M, 'The Migration of a Name: Reflections on Alexander in Africa', *Cultural Anthropology*, Vol 2, No 2 (May, 1987) 235–254.

[10] Ibid, 240.

[11] Ibid, 240.

The other difficulty is that Maddox does not consider the incorporation of Captain Cook into the cosmological structure of the dreaming. None of the stories that he studies can be properly described as dreaming and his use of myth is only intended to distinguish the Captain Cook stories he has studied from the historical figure (the "real" Captain Cook). *Too Many Captain Cooks* departs from, for instance, the structure of the Cook/Kelly story because Captain Cook is an ancestor. Moreover, none of Maddox's stories present Captain Cook in a good light nor as intimately connected to Aboriginal people. We have to consider the role the dreaming plays in order to understand *Too Many Captain Cooks*. My general contention is that a reading of *Too Many Captain Cooks* is possible without an intimate knowledge of the dreaming, but there are attributes of the dreaming that contribute to my proposed reading. *Too Many Captain Cooks* is more than cosmological in the strict sense because of its use of western symbolism, especially the second part of the story which is a familiar account of the impact of colonisation on Aboriginal people. That is to say, it is possible to read the Rembarrnga dreaming from a perspective of what it says about "us" as coloniser (as "we" are part of the object of the story), the Anglo-European colonial project and what it tells us about the Rembarrnga response to colonisation.

Those key attributes of the dreaming which contribute to my reading are as follows. The dreaming is a complex institution. It is a time when law is made and is a sacred and heroic time when human and nature came to be as they are.[12] However, neither time nor history as we understand it is involved in this meaning.[13] As the celebrated anthropologist W E H Stanner puts it, 'one cannot fix the dreaming in time: it was, and is, everywhen'.[14] The dreaming infuses the past, present and the future. Thus, there is an important continuity between the dreaming and the here-and-now. The dreaming also talks about what life is and what it can be[15] and it is for this reason often associated with law. The ancestor through intentional and unintentional acts lays the foundations of the law which is revealed in the dreaming. In Stanner's words, the dreaming is:

[12] Stanner W E H, 'Dreaming' in *White Man Got No Dreaming* (Canberra, 1979), 23.

[13] Ibid, 24.

[14] Ibid.

[15] Ibid, 29.

> A kind of narrative of things that once happened; a kind of charter of things that can still happen; and a kind of *logos* or principle of order transcending everything significant for Aboriginal Man.[16]

The dreaming is also, importantly, a framework for care and control of country.[17] It is through the dreaming that Aboriginal people have rights and responsibilities in relation to land and a chief complaint about colonial interference is that it prevents Aboriginal people from caring for country.[18] Indeed Hobbles, an Aboriginal man from the Yarralin settlement in the Northern Territory, ironically uses the expression 'Captain Cook's "law"' to characterise the progressive supplanting of an Aboriginal way of caring for country with an Anglo-European way.[19] Captain Cook's law is not solely a reference to western law in an institutional sense, though law in this sense is undoubtedly an important factor in Hobbles' complaint about the loss of control of country, it is also a broader reference to what Hobbles sees as destructive environmental practices (Anglo-European ways of caring for country).

A preliminary reading to lay the groundwork for what follows is that the representation of Captain Cook as law man is as simple and as complex as trying to share in and control what is seen by the Rembarrnga as the source of power of the coloniser. Captain Cook, as we shall see, is a metonym in both law and the Anglo-European public consciousness for sovereignty. Sovereignty has become a thing of significance for Aboriginal people. The making of Captain Cook as law man attempts to close the gap between dreaming — the time things of significance came into being for Aboriginal people — and the present. It is through Rembarrnga law via *Too Many Captain Cooks* that Captain Cook, as a metonym for sovereignty, is claimed as a significant property of the Rembarrnga.

[16] Ibid, 24.

[17] See Ingold T, 'Hunter-Gathering as a Way of Perceiving the Environment'in *The Perception of the Environment: Essays on Livelihood, Dwelling and Skill* (London and New York, 2000), 53; Munn N D, 'The Transformation of Subjects into Objects in Walbiri and Pitjantjatjara Myth' in Berndt R M (ed), *Australian Anthropology: Modern Studies in the Social Anthropology of the Australian Aborigines* (Perth, Western Australia, 1970), 146 and 148; Myers F, *Pintupi Country, Pintupi Self: Sentiment, Place, and Politics among Western Desert Aborigines* (Berkeley, 1986), 49–50.

[18] Myers, ibid, 49–50.

[19] Rose D B, *Dingo Makes us Human: Life and Land in an Australian Aboriginal Culture* (Cambridge, 1992), 189.

There is an extraordinarily paradoxical consequence caused by the incorporation of Captain Cook into the dreaming. The Rembarrnga Captain Cook *as* metonym for sovereignty is prior to Anglo-European sovereignty. To paraphrase Wainburranga here: Rembarrnga know the real Captain Cook; Rembarrnga know and respect only one Captain Cook; and Captain Cook is an ancestor from millions of years ago — from the time of the dreaming — not from 200 years ago. *Too Many Captain Cooks* is also a contestation of Anglo-European sovereignty in Australia which is intimated in Wainburranga's words that, '*other people started thinking they could make Captain Cook another way'* and 'no one can change our law, no one can change our culture ... we have the story of Captain Cook'. In this regard, *Too Many Captain Cooks* is both an assertion and a contestation of sovereignty — it is a strategy of power and not a metaphor for political power. To flesh out this argument, we must look more closely at the "object" of the story: the "historical" Captain James Cook.

1.2. Captain James Cook — His Majesty's Object

On a fateful day, 22 August 1770, Captain James Cook came to what he later called 'Possession Island' and took possession of the east coast of the continent now known as Australia on behalf of King George III. Captain Cook, the renowned English explorer made three South Pacific voyages (1768–71, 1772–5, 1776–80). It was during his first voyage that he navigated Australia in the ship named the Endeavour. Captain Cook had made a number of other declarations purporting to take possession of the east coast on the course of that voyage. However, the significance of that day was that Captain Cook had traversed the entire length of the east coast of the continent. Captain Cook records:

> We saw a number of People upon this Island arm'd in the same manner as all others we have seen except one man who had a bow and a bundle of Arrows the first we have seen on this coast. From the appearance of these People we expected they would have opposed our landing but as we approached the Shore they all made off and left us in peaceable possession of as much of the Island as served our purpose. After landing I went upon the highest hill which however was of no great height, yet not less than twice or thrice the height of the Ships Mast heads but I could see from it no land between SW and WSW so that I did not doubt

> but there was a passage, I could see plainly that the Lands laying to the NW of the passage were composed of a number of Islands of various extent both for height and circuit reigned one behind another as far to the Northward and Westward as I could see which could not be less than 12 or 14 Leagues. Having satisfied myself of the great Probability of a Passage, thro' which I intend going with the Ship and therefore may land no more upon this Eastern Coast of New Holland and on the Western side I can make no new discovery the honour of which belongs to the Dutch Navigators ... but the Eastern Coast from the Latitude of 38 South down to this place I am confident was never seen or visited by any European before us and notwithstanding I had in the name of his Majesty taken possession of several places upon this coast I now once more hoisted English Coulers and in the Name of his Majesty King George the Third took possession of the whole Eastern Coast from the above Latitude down to this place by the Name of New South Wales together with all the Bays, Harbours, Rivers and Islands situate upon the said coast after which we fired three Volleys for small Arms which were Answered by the like number from the Ship this done we set out for the Ship ... We saw on all the Adjacent Lands and Islands a great number of smooks [smokes] a certain sign that they are inhabited and we have dayly seen smooks on every part of the coast we lately been upon.[20]

While discredited in international law because it was prone to abuse, 'discovery' of a territory nevertheless was considered sufficient to provide the European sovereign with title.[21] It is at least clear from Cook's journals that he thinks discovery is sufficient. In *Mabo*, sovereignty is described simply as an 'act of state'[22] and while there is seemingly confusion amongst the judges over what "act" actually constituted the sovereign event, Captain Cook certainly forms part of the sovereign pantheon of

[20] Cook J, 'Cook's Journal: Daily Entries 22 August 1770', `http://southseas.nla.gov.au/journals/cook/17700822.html`; `http://southseas.nla.gov.au` (16 May 2007).

[21] Anghie A, *Imperialism, Sovereignty and the Making of International Law* (Cambridge, 2004), 82.

[22] Brennan J, *Mabo*, above n.3, [31].

acts. At the very least he provides continuity between discovery and that later settlement of the colony by Captain Arthur Phillip and the First Fleet of 1788. Captain Cook, I suggest, is a metonym of sovereignty in law.

In the Anglo-European public consciousness he sits, some say inaccurately, alongside Captain Arthur Phillip of the first fleet as an 'outstanding figure in the founding of Australia'.[23] Chris Healy argues that we should not assume that the, now exalted, place of Captain Cook in Australian history has always been a continuous one (from past to present). There was an active movement amongst an elite segment of the Australian population to turn Captain Cook into an identifiable "Australian" figure within the wider framework of a history encompassed by Europe.[24] In Healy's words,

> Those who believed passionately in Cook wanted his very name to perform a general public role as variously European, British, imperial, visionary and nationalist. Cook was to provide both a structure of historical time and a point of genesis which would serve to mark the end of empty time and the beginning of continuous historical time in Australia.[25]

We can think about the relevance of Jackson's concept of the transmigration of a name in relation to Captain Cook's place in Anglo-European Australian history. After all, as Healy points out, he lived and died somewhere else.[26] There are some interesting parallels between the historical and dreaming figure of Captain Cook which are worth drawing out here. In dreaming stories ancestors have generative or constitutive powers. They mark out sites as significant — hills, salt lakes, trees — by metamorphosing into these geological forms of the landscape in their travels.[27] 'Ever present in these forms, their movements are congealed

[23] See Maddox, 'Myth, History and a Sense of Oneself', above n.6, 13 and 24.

[24] Healy C, *From the Ruins of Colonisation: History as Social Memory* (Cambridge, 1997), 7.

[25] Ibid, 30.

[26] Ibid.

[27] Myers F, *Pintupi Country, Pintupi Self: Sentiment, Place, and Politics among Western Desert Aborigines*, above n.17, 49–50.

in perpetuity'.[28] Discovery and cartography appears to fit neatly with the world constitutive power of dreaming. The great navigators, of whom Christopher Columbus and Captain Cook have become household names, resemble dreaming ancestors who wrought radical transformations on a territory and the world. Like a dreaming ancestor Captain Cook marked the east coast of Australia with cultural significance for Anglo-European and Aboriginal alike, namely it became New South Wales and a British possession. By the 1930s in Australia there had been a spread of historical inscriptions marking Captain Cook's landing places. As Healy puts it:

> Particularly in the case of Cook, the memorialising of landing places meant anchoring travelling deeds as if they were generative acts, as if an emergency landing at Cooktown was actually connected to the place which it had become. In other words, these were not acts of preserving memories in place but of memorialising events, which were then to be remembered in a place other than their performance.[29]

The use of "object" in the subtitle of this section is purposeful. We can no longer conceive of Captain Cook as a "real" person in the ordinary sense of the word. He and his name have been deployed by Anglo-European Australians in a way similar to the way in which Alexander the Great has been deployed across history and cultures. It is left open to us to ask — who deified Captain Cook, or to put it another way, who is responsible for the apotheosis of Captain Cook? There are numerous historical and anthropological studies that argue Captain Cook was deified by the "natives" he encountered on his voyages, much like the raising of Captain Cook to the status of ancestor by the Rembarrnga. Marshall Sahlins' anthropological study on the Hawaiian's mistaking Captain Cook for the god 'Lono' is the most well known.[30] Sahlins' thesis has been attacked by Gananath Obeyesekere, arguing that it was actually the English (such as missionaries and anthropologists) who raised Captain Cook to

[28] Ingold T, 'Hunter-Gathering as a Way of Perceiving the Environment', above n.17, 53.

[29] Healy, *From the Ruins of Colonisation: History as Social Memory*, above n.24, 36.

[30] Sahlins M, *How "Natives" Think About Captain Cook, for Example* (Chicago, 1995).

the status of a god or spread his reputation as god-like.[31] This perception of Captain Cook supported the sense of destiny as "Teacher of Nations" that the British felt in the colonial context.[32]

Obeyesekere's perspective is supported by Kathleen Wilson's study of what she describes as the apotheosis of Captain Cook in England during the eighteenth century. There were numerous representations of his achievements from publications, biographies, plays, poetry and paintings. These representations helped recuperate, Wilson argues, 'British political and imperial authority, rescue the national reputation for liberty and restore faith in the superiority of the English character' and the English genius for discovery and exploration.[33] As Wilson points out, Captain Cook reached a heroic stature in English national consciousness that few figures before or since have matched and his continued importance is assured as study after study assesses the impact of his legacy.[34]

The debate over the genesis of Captain Cook's god-like status suggests that Captain Cook meets Max Weber's notion of charisma. Charisma is rooted in some quality or character not accessible to everybody.[35] It may be that the "real" Captain Cook was undeserving of such a reputation but, at least, in the public imagination his feats and character were considered deserving of lofty accolades. Captain Cook had 'superior abilities, judgment and discipline' as well as 'humble origins as a Yorkshire husbandman's son', was 'auto-didactic' (he taught himself mathematics and astronomy) and exhibited 'humility'. In Wilson's words, 'all become inextricable parts of his heroic character'.[36]

It is Weber's linkage of charisma with political or religious authority that is most important here. Charismatic authority is a spontaneous form of authority that can be contrasted with, at least in Weber's view, institutional authority. Weber has been criticised for sometimes reducing

[31] Obeyesekere G, *The Apotheosis of Captain Cook: European Mythmaking in the Pacific* (Princeton, 1992).

[32] Wilson K, *The Island Race: Englishness, Empire and Gender in the Eighteenth Century* (New York, 2003), 91.

[33] Ibid.

[34] Ibid.

[35] Weber M, 'The Sociology of Charismatic Authority' in Heydebrand W, *Sociological Writings* (New York, 1994), 254.

[36] Wilson, *The Island Race: Englishness, Empire and Gender in the Eighteenth Century*, above n.32, 6.

charismatic authority to, in Pierre Bourdieu's words, 'a spontaneously generated product of inspiration'.[37] However, as Bourdieu points out, while underdeveloped in Weber's thought, he did recognise the work carried out by specialist agents or an elite class, such as a priestly caste, in sustaining, regulating and regularising the authority of the charismatic figure, usually, after the figure's death. [38] This goes someway towards explaining the longevity of some figures considered to be charismatic.

This capturing, so to speak, of the charismatic authority by specialist agents is a form of institutionalisation. In Bourdieu's words:

> And the principle of this institutionalisation consists, for Weber, in the process whereby charisma detaches itself from the person of the prophet to attach itself to the institution and, more precisely, to a specific *function*: 'the process of transferring such sacredness which derives from charisma to the institution as such ... is characteristic of all processes of Church-formation and constitutes their specific essence.[39]

There is a key distinction that Bourdieu draws between Church-formation and the proliferation of the sect (another type of "institution" that also claims the charismatic figure as its property). Putting aside issues of authoritativeness, this suggests that no one institution has a monopoly on the deployment of a charismatic figure.

Returning to Bourdieu's account of the institutionalisation of charismatic authority, it is possible to substitute the term Church-formation with that of Nation/State-formation. The discussion above of the deployment of Captain Cook by elites in Australia supports this substitution. I suggest, however, that the judiciary are also capable of falling within Weber's and Bourdieu's concept of specialist agents and that the High Court's, albeit ambivalent, equation of Captain Cook with sovereignty performs a similar role in Australian law that Captain Cook does for Australian history. Captain Cook provides a point of genesis of a continuous sovereignty. The Captain Cook of Australian law, like Captain Cook the dreaming ancestor, is also a strategy of sovereign power.

[37] Bourdieu P, 'Legitimation and Structured Interests in Weber's Sociology of Religion' in Lash S and Whimster S, *Max Weber: Rationality and Modernity* (London, 1987), 119.

[38] Ibid.

[39] Ibid, 135.

2. Mimesis and Sovereign Power

2.1. Mimesis and Power

The argument of this paper that sovereignty in Australia is mimetic invokes the concept of mimesis. My reading of *Too Many Captain Cooks* largely rests on this concept. What, then, is mimesis? In very simple terms mimesis refers to the mimicry or the copying of something and the dialectical relationship (the act of mimicry) between an original and its copy.

The critical point I seek to make in this section is that mimesis is about power; its generation, control, manipulation as well as its questioning and contestation. I have already suggested that the Rembarrnga portrayal of Captain Cook is a strategy of power, more precisely, a strategy of sovereign power that contests the Captain Cook of the colonial project. Thus, there is in the Rembarrnga example a power play between two Captain Cooks. In conventional thinking, best exemplified by Maddox, that power play is between an authoritative Captain Cook and a figurative or metaphoric Captain Cook. On my reading this power play is inverted in the Rembarrnga dreaming. The Rembarrnga version of Captain Cook is the authoritative version and the Captain Cooks that follow, albeit powerful, are impostors. How are we to account for two very different versions of the dialectical relationship between the original and the copy?

There are, in my view, seven interrelated aspects of the dialectical relationship between the original and the copy which provide a theoretical framework for understanding these two versions of the power play between Captain Cook and Captain Cook. The first aspect is the *power of mimesis*. That is, the copy shares in and takes power from the original. The second aspect of the relationship between the original and the copy is the issue of *imperfect copies*. The issue can be posed as a question — how exact does a copy have to be in order to be properly called a copy? The third aspect is *communicative*. Mimesis or, more appropriately, mimicry and mime emerged as a communicative strategy in the colonial context. The fourth aspect is *temporality*. There are two aspects to temporality if we locate it in the colonial context. The first is the construction of authority (the original) in the colonial context. The second can be put in the form of a question — what comes first or who is mimicking whom? The question indicates that there is the possibility of temporal slippage and cyclical play between the copy and the original.

The fifth relationship is *ambivalence* and it concerns the affect that the copy can have on the original. The copy threatens to undermine the authority or authenticity of the original. The sixth aspect is *contestation* or *conflict*. While I propose to develop this aspect in greater depth in the next section, contestation can be broken into two aspects: the disrupting effects of ambivalence and the actual contestation over the authority, or the authoritative nature, of the "original" object. The seventh and final aspect of the mimetic relationship that I have identified is *reciprocity*, which I suggest adheres or is inherent in mimesis. I will say nothing more of this aspect as I will draw out the reciprocal nature of mimesis when I address contestation.

In relation to the first aspect, I am interested here in Taussig's discussion of magical practice as a form of mimesis because it reveals so well the power play of mimesis. Magical practice took on a new type of mimetic quality in the colonial context, in which images of Europeans are incorporated into the craft of magic. The Cuna Indians of the San Blas Islands off Panama, for example, had carved wooden figurines pivotal to curing in the likeness of Europeans.[40] In the late 1940s one observer even noticed a figure in the likeness of General Douglas MacArthur.[41] Exactly when this transformation occurred is unclear, but what is clear is that at a certain point the healing figurines no longer looked like either Indians or demons.

The wooden figurines are an important aid to healing. For example, these figurines or, more importantly, the spirits that they represent search for an abducted soul of a sick person.[42] In one healing of a woman in obstructed labour the medicine man took the wooden figurines and sang to them the following: 'the medicine man gives you a living soul, the medicine man changes for you your soul, all like replicas, all like twin figures'.[43] Taussig sees an intrinsic connection between mimesis — the act of copying or replicating something — and the magic hinted at in the use and function of the wooden figurines. As Taussig puts it:

[40] Taussig M, *Mimesis and Alterity: A Particular History of the Senses* (New York, 1993), 7–8.

[41] Ibid, 10.

[42] Ibid, 9.

[43] Ibid, 7.

> Note the replicas. Note the magical, the soulful power that derives from replication. For this is where we must begin; with the magical power of replication, *the image affecting what it is an image of, wherein the representation shares in or takes power from the represented.*[44]

(I will put aside for the moment that the example also intimates that Europeans embody power or, saying the same thing in a slightly different way, are an object or symbol of power.) The example suggests that the power of the copy is derivative; that is, its power is generated from its association with the original.

The wooden figurines invoke James Frazer's two species of magic: the magic of similarity and the magic of contact or contagion. The first is based on the principle that 'like produces like' or an 'effect resembles its cause'.[45] The magic of similarity is best thought about by taking voodoo as an example as it is well represented in western popular culture. In voodoo an effigy in the image of someone is made 'in the belief that just as the image suffers, so does the man, and that when it perishes he must die'.[46] In Frazer's words, 'the magician infers that he can produce any effect he desires merely by imitating it'.[47] The magic of contact or contagion is based on the principle that 'things which have once been in contact with each other continue to act on each other after the physical contact has been severed'.[48] It uses items of clothing or body parts such as hair, nails, teeth and so on, to be magically acted upon.[49] While not requiring a more exact copy like the magic of similarity, it works on the same principle that there is a connection between one thing and another. In both examples as G E R Lloyd puts it,

> [Magic's] general aim is similar to that of applied science, to control events, and one of the means whereby it hopes to achieve this is using the links which it believes may be formed between things by their similarities.[50]

[44] Emphasis added. Ibid, 7–8.
[45] Frazer in ibid, 47.
[46] Frazer in ibid, 48.
[47] Frazer in ibid, 47.
[48] Frazer in ibid.
[49] Ibid, 53.
[50] Ibid, 49.

Taussig's claim that the representation shares in or takes power from the represented says very little about the type or, even, the quality of the effect that the copy has on the copied and vice versa. The magic of similarity — like produces like — would suggest that in circumstances where the copy was similar enough to the "thing" that it replicates the copy takes on all the same attributes. Copyright law is based on this very premise. A copy of the 'text', to use legal terminology, derives its power from the original text and, indeed, affects the power and appeal of the original text as commodity. However, it is unlikely to be the case across the board that the copy will take on the same attributes as the original, bringing us to the problem of imperfect copies (the second aspect). The species of magic Frazer calls the magic of contact already alerts us to one element of this problem of imperfect copies, given that the link or resemblance between the copy and the original is more remote. The second element is that imitation may not, despite Frazer's assumption otherwise in relation to the magician's goal, produce the desired effect. The problem of imperfect copies is a problem of both form and effectiveness.

I will address these complexities of copying by discussing Walter Benjamin's spectacular paper on art in the age of mechanical reproduction. The paper concerns the effect that mechanical reproduction has on the work of art and, in particular, the affect that reproduction has on the work of art's aura. A critical reading of the essay brings the problem of imperfect copies into sharp relief. Benjamin uses the word 'aura' to refer to the sense of awe and reverence one experiences in the presence of unique works of art.[51] With the advent of art's mechanical reproducibility, and the development of forms of art in which there is no actual original (such as film), the experience of art could be freed from place and ritual and instead brought under the gaze and control of a wider audience, leading to a shattering of the aura of the work of art.[52]

It might be that Benjamin's theory is more appropriate to art (such as film) in which there is no actual original. Classical works of art (such as paintings) can only ever be imperfectly copied via mechanical or, these days, digital reproduction (such as in the form of a poster, a postcard or in a publication). Whereas, if a painting were to be expertly copied it would

[51] Benjamin W, 'The Work of Art in the Age of Mechanical Reproduction' in Arendt H (ed), *Illuminations* (London, 1970), 215.

[52] Ibid.

be evaluated as either a forgery or fake, regardless of how perfect the copy is.[53] The original preserves its authority and authenticity.[54] Therefore, it is arguable that mechanical reproducibility only serves to enhance the aura of the painting; after all there is only "one" Mona Lisa. As Benjamin puts it: 'even the most perfect reproduction of a work of art is lacking in one element: its presence in time and space, its unique existence at the place where it happens to be'.[55] This view accords most closely to the conventional view stated above concerning the copying of Captain Cook in Aboriginal stories. Captain Cook as ancestor is an "imperfect" copy or, as Maddox would have it, is not authoritative.

This is not to say, however, that the aura of the original painting is not troubled in some way by the spreading of multiple copies. 'The situations into which the product of mechanical reproduction can be brought', Benjamin says, 'may not touch the actual work of art, yet the quality of its presence is always depreciated'.[56] For Benjamin the aura was not inherent in the object but rather was generated by its control (via ownership, history and tradition), in particular the control over access to it through its restricted exhibition. Reproducibility detaches the work of art from the domain of tradition and its control putting the copy, to paraphrase Benjamin, into situations which would be out of reach for the original itself. It may be that reproductions of a painting invoking the presence of the original are used in ways that were never intended, thereby undermining the tradition and historical associations of the original work of art. In Benjamin's words, 'by making many reproductions it substitutes a plurality of copies for a unique existence'.[57]

Shifting focus a little I want to discuss the communicative aspect of mimesis, which is the third aspect of the relationship I have identified between the original and the copy or, perhaps more appropriately in this context, the original and the mimic. First contact provides a number of rich examples of mimicry and mime — side by side with exchange — as one of the central modes of communication in the colonial context. Charles Darwin's famous expedition on the Beagle in 1832 provides the first account of mimetic "exchanges" between the Europeans forming the

[53] Ibid, 214.
[54] Ibid.
[55] Ibid.
[56] Ibid, 214–215.
[57] Ibid, 215.

expedition party and the people of Tierra del Fuego (the Fuegians) during, near, first contact. Darwin's account consists of observations of the differences that he sees between 'savage' and 'civilised man' and is littered with disparaging comments about the Fuegians' language ('like a man trying to clear his throat'), dress and cultural depravity.[58]

But, the Fuegians are, in Darwin's words, 'excellent mimics: as often as we coughed or yawned or made any odd motion, they immediately imitated us'.[59] One should not mistakenly assume that Darwin's comment that the Fuegians are excellent mimics is intended to be complimentary. Mimicry has long been intimately associated with primitiveness or infancy in European thought.[60] It appears, however, that Darwin is guilty of a form of mimetic myopia. Captain Fitz Roy's account of the exchange reveals something wholly missing from Darwin's account.

> They expressed satisfaction or good will by rubbing or patting their own, and then our bodies; and were highly pleased by the antics of a man belonging to the boat's crew, who danced well and was a good mimic.[61]

The second account of communicative mimicry is taken from Mick Leahy, an Australian gold prospector discussing the exchanges with people from the highlands who had never before had contact with whites. Leahy says:

> We told the [highland] natives of our intention by signs and asked them to come down the next morning and show us the way. This was accomplished by leaning the head on one hand and closing the eyes — gestures of sleeping; pointing to the ground, to indicate this place; then pointing to the east, with a rising gesture — "sun he come up", and then pointing off down the creek, looking down for a trail and shaking our heads. The natives got it at once, and gave us to understand that they would be on hand. *Pantomime serves surprisingly well for conversation when you have to depend on it.*[62]

[58] Darwin in Taussig, *Mimesis and Alterity*, above n.40, 74.

[59] Darwin in ibid.

[60] Ibid, 81 and Benjamin W, 'On the Mimetic Faculty' in Demetz P (ed), *Reflections: Essays, Aphorisms, Autobiographical Writings* (New York, 1978).

[61] Fitz Roy in ibid, 76.

[62] Emphasis added. Leahy in ibid, 78.

The communicative aspect of mimicry and mime appears to have passed Darwin by, but Leahy recognises its fundamental pragmatic importance. Surely the compulsion to mimic was a very important communicative strategy, which is missed by Darwin's reduction of mimicry to mere automotive primitive gestures. Mimicry is here engaged as a translative and communicative strategy to bridge linguistic, cultural and other gulfs to recognition. If mimesis also functions as a communicative strategy this raises two possibilities if we bring the discussion back to *Too Many Captain Cooks.* First, Captain Cook the ancestor could presumably be thought to stand for something already existing within Rembarrnga cosmology, which without its conversion into a recognisable symbol would be lost on its intended audience. This undoubtedly raises question marks over whether the copy is a copy at all in the true sense. Secondly, it is also possible that the Rembarrnga have recognised in Captain Cook something underscoring Anglo-European colonial power and have sought via Captain Cook's incorporation into the dreaming to meet this power head on and to contest its control by Anglo-European Australia. If on one reading of Carl Schmitt, the German constitutional jurist, sovereignty is the capacity to decide,[63] the assertion of sovereignty in *Too Many Captain Cooks* is surely a bit of both. *Too Many Captain Cooks*, I contend, is an expression of a prior sovereignty manifest *in* the dreaming — the Aboriginal law for care and control of country indeed, the capacity to decide for country — as well as a contestation of the Anglo-European stranglehold on sovereign power in Australia.

We can also gauge the issue of temporality, the fourth aspect of the dialectical relationship, in Darwin's and Fitz Roy's accounts of mimicry. I suggested above that there are two parts to the aspect of temporality and the first is the construction of authority (the original) in the colonial context. Darwin provides us with a striking snapshot of colonialism. The enormous importance of Darwin's account is that in it we witness the emergence of an original in the colonial context ('they', as Darwin asserts, 'imitated us').[64] Added to this, Darwin's account incisively represents the investment of colonialist selfhood through the prejudiced observation of

[63] Schmitt C, *Political Theology: Four Chapters on the Concept of Sovereignty* (Cambridge, 1985).

[64] Taussig, *Mimesis and Alterity*, above n.40, 79.

primitives so that we also are witness to the emergence *by* an original.[65] In Taussig's words, 'civilization takes measure of its differences through its reflection in primitives'.[66] In a similar vein, Bhabha argues that the English have no authority of their own but gain their authority only in the colonial context on the premise of colonial difference.[67]

I want to stay with Bhabha here as he provides a useful way to think about this complex issue of the generation of colonial power and authority, already indicated in Darwin's observation of the Fuegians. There were, if it is possible to talk about colonialism in the past tense, two broad types of colonial power and knowledge concerning the colonial "subject", which we can place under the umbrella terms of scientific racial difference and humanist universalism. Scientific racial difference and humanist universalism represent two extremes of a spectrum of knowledge and power in the colonial frontier. What they have in common is that the operation of the two types of power and knowledge produces discriminatory differences.

Scientific racial difference, in its ascendency in the eighteenth century, was rooted in the classification of people into races differently positioned on a hierarchical scale based on categories such as intelligence and cultural sophistication. Scientific models of craniometry, for example, were used to measure the intelligence of 'Man' and "proved" that Africans, Asians and Aboriginal peoples were racially inferior.[68] Scientific racial difference was a pure form of discriminatory difference in which the differences between races were seen to be immutable and, for those at the bottom of the scale, nothing could, to invoke the famous Privy Council case on *terra nullius*, bridge the gulf.[69] Humanist universalism is based on the belief in (or desire for?) an underlying unity in the human experience. As Stewart Motha puts it, 'the other is transported/transferred into an imagined *we*, a community wrought on the back of the erasure of particularity in the name of a universal, abstract commonality'.[70] It is humanist universalism

[65] Ibid.

[66] Ibid, 79.

[67] Mohanram R, 'The Postcolonial Critic' in Wilson M and Yeatman A (eds), *Justice and Identity: Antipodean Practices* (Sydney, 1995), 189.

[68] Banton M, *Racial Theories* (Cambridge, 1998) and Gould S J, *The Mismeasure of Man* (London, 1992).

[69] Lord Sumner cited in Brennan J, *Mabo*, above n.3, [38].

[70] Motha S, 'Mabo: Encountering the Epistemic Limit of the Recognition of "Difference"' 7 *Griffith L. Rev.* 79 (1989), 86.

that is my principal interest here and its linkage, as its epistemological backbone, to colonial mimicry. Bhabha argues that 'mimicry emerges as one of the most elusive and effective strategies of colonial power and knowledge'.[71]

Colonial mimicry is the 'desire for a reformed, recognisable Other, as a subject of a difference that is almost the same, but not quite'.[72] By invoking sameness and difference in the same sentence there is an apparent contradiction in Bhabha's definition of colonial mimicry. The contradiction can be resolved if we think of the project of colonialism as the simultaneous desire for reformation of the Other — the civilising mission — and subjugation or domination, two dual bases of colonial rule. In this respect, colonial mimicry is a project of (ambivalent) assimilation that is most powerfully exemplified by two examples. First, ambivalent assimilation is perfectly illuminated in missionary thinking. In Christian universality all peoples are always *potential* children of God, but are, to paraphrase Bhabha, not quite. With tutorage and pastoral care (the European "mission" and the institutional manifestation of domination and subjugation) heathens can be reformed and assimilated into the flock of God's children.

Secondly, this same ambivalent assimilation or, to put it another way, discriminatory universality is evident in Francisco De Vitoria's international law scholarship on the rights of the Spanish in relation to South American Indians and Indian territory. In Anthony Anghie's words:

> According to Vitoria, Indian personality has two characteristics. First, the Indians belong to the universal realm like the Spanish and all other human beings, because Vitoria asserts, they have the facility of reason and hence a means of ascertaining *jus gentium* which is universally binding. Secondly, however, the Indian is very different because the Indian's specific social and cultural practices are at variance from the practices required by the universal norms — which in effect are Spanish practices — and which are applicable to both Indian and Spaniard. Thus the Indian is schizophrenic, both alike and unlike the Spaniard.[73]

[71] Bhabha Homi K, *The Location of Culture* (New York, 1994), 85.
[72] Bhahba, ibid, 86.
[73] Anghie, *Imperialism, Sovereignty and the Making of International Law*, above n.21, 22.

The application of *jus gentium* to the Indians meant that they were obliged by natural law to allow the Spanish to 'travel' and 'sojourn' in the land of the Indians, whereas because of cultural differences (such as their status as heathens) the Indians are effectively excluded from the realm of sovereignty. As heathens they are unable to engage in a 'just' war, a sovereign's right, in circumstances of Spanish incursions on Indian territory.[74] Both are examples of the production of discriminatory identities (an imperfect copy) that secures the "pure" and the "original" or, more appropriately in this context, the universal.

The second part of the aspect of temporality can also be found in Darwin's and Fitz Roy's accounts of mimicry. It is already evident in Darwin's thoughts that the colonial context is pregnant with structural power relations and one of the generators of power relations is different layers of mimesis. If we take Darwin to be a symbol of western colonial thinking about the "savage" we see only one side of the mimetic relationship (this one-sidedness is itself a product of colonial thinking). Fitz Roy provides us with an alternative way to think about the mimetic relationship; the emergence of the original and by an original is already swimming in the shallows of temporal problems. If we compare Fitz Roy's account with Darwin's we are left with the question posed by Taussig — 'who is mimicking whom, the sailor or the savage?'.[75] This is another way of asking the recurrent question, what comes first, the original or the mimic? Both questions highlight the problem of temporal slippage or blurring.

The problem of temporality naturally leads us to the ambivalence that always threatens to engulf the original when it is copied. Ambivalence is the fifth aspect of the relationship between the copy and the original. Ambivalence can also be called the menace of mimicry. The desire of a reformed and recognised Other in the colonial project contains the seeds of the undermining of colonial power and authority. This undermining is engendered by what Bhabha calls hybridity, which is the mixing that occurs between cultures so that binaries like colonised/coloniser and savage/civilised become unstable. One need only think of the colonial anxiety that was caused by the miscegenation of races and the fear that it would cause a dilution of the European race, as an example of hybridity. The

[74] Ibid, 20.
[75] Taussig, *Mimesis and Alterity*, above n.40, 76–77.

mixing of races also caused immense confusion for colonial administrators over how to categorise offspring.

The menace of mimicry is what happens to colonial authority and power when the Other, recognised *as* Other, takes up a text, symbol, sign or discourse of colonial power (now a hybrid). As Bhabha puts it, colonial presence 'is always ambivalent, split between its appearance as original and authoritative and its articulation as repetition and difference'.[76] The power of the menace of mimicry to undermine the authority of colonial power is revealed by the questioning of the Bible by Indian converts outside Delhi in the early 1800s. The equation of the Bible with the English, one strategy upon which colonial power rested is put to question by Indian converts.

> The native questions quite literally turn the origin of the book into an enigma. First: *how can the word of God come from the flesh-eating mouths of the English?* — a question that faces the unitary and universalist assumption of authority with the cultural difference of its historical moment of enunciation. And later: *how can it be the European Book, when we believe that it is God's gift to us? He sent it to us at Hurdwar.*[77]

The questions go to the origins of the Bible, particularly its embedded tradition in Europe. Let me return to the problem identified in Benjamin's thought about the original work of art. There is only one Bible and, by all measures, it is the authoritative text. However, the questions raised by the Indian converts are made possible by the translation of the Bible — its reproducibility — into local dialects, which not only enhances its accessibility but estranges the word of God from sole association with the English. The unique place of the Bible in Europe/Britain is undermined by the menace of mimicry.

2.2. Sovereign Power — Girard's Mimetic Desire and Hobbes' Leviathan Motivation

There are two essential points that I wish to make in this section as a lead into a closer look at *Mabo*, the 1992 landmark judgment in Australian law that recognised that Aboriginal people possessed a form of proprietary

[76] Bhabha, *The Location of Culture*, above n.71, 107.
[77] Emphasis in the original. Ibid, 116.

rights called ‘native title’. *Mabo* is, as we will see, the High Court’s response to this contest over sovereignty. First, drawing on Girard’s theory of mimetic desire I will show how contestation over an object (like sovereignty) causes societal crisis. This point develops in greater depth the ambivalence between the original and the copy caused by the menace of mimicry. Second, I will illustrate that the mimetic desire underlying societal crisis is manifested in institutional form, as sovereign power, in Hobbes’ *Leviathan*. Hobbes’ *Leviathan* is the enormously influential text on the nature of western sovereignty. In this respect, I will turn the gaze of this paper towards, what I suggest, is the sovereign behaviour of the High Court in Australia.

I want to first outline the story of the ‘spirit boat’.[78] In my view the spirit boat exemplifies the use of mimesis as a strategy of power, as a precursor to contestation, in a way which is not apparent on the face of the Rembarrnga dreaming. The story of the ‘spirit boat’, relayed to an anthropologist by Choco Indians, is about a Shaman who was ‘frightened speechless’ by a visitation by the spirits of white men and who, in a daring move, decided to capture them to add to his stable of spirit helpers.[79] It turns out that the visitation in question is an event where the Shaman sees a boat of white men while in a canoe on the Congo River.[80] The Shaman’s grandson describes the event:

> We saw a boat of many colors, luminous with pure gringos aboard. It sounded its horn and we, in the canoe, hauling, hauling, trying to catch up to the boat. We wanted to sleep alongside it but the boat moved out to sea, escaping us. Then we smelled gasoline. Our vision could no longer stand the fumes and [the shaman said]: “Let’s go back. This is not a boat. This is a thing of the devil”.[81]

The Indians in the canoe became violently sick and consumed by fear.[82] Once they had managed to get home they prepared for a healing ritual. Instead of a ‘defensive ritual’ which would have healed the party of

[78] Taussig, *Mimesis and Alterity*, above n.40, 14.
[79] Ibid.
[80] Ibid.
[81] Ibid.
[82] Ibid, 15.

Indians who witnessed the spirit boat, the Shaman decided to capture the gringo spirit-crew for himself and makes a copy of them and the boat.[83] The spirit boat is a strategy of power where the Shaman captures the gringo spirit-crew in what is an offensive rather than a defensive gesture.

Girard's mimesis reworks the dialectical relationship between the copy (called the 'subject') and the original (called the 'model'). Girard structures this relationship — in a sense fuels it — with the concept of rivalry. We are presented with the fascinating, even startling, claim that the subject's attention is drawn to the object because the model desires it; that is to say, desire is mediated. Livingston notes that Girard is using desire here as *le désir selon l'Autre* or 'desiring according to the Other', as opposed to *selon soi* or 'desire that is a spontaneous and autonomous manifestation of an individual's wants or preferences'.[84] It is worth quoting Girard on mimetic desire in full here:

> Rivalry does not arise because of the fortuitous convergence of two desires on a single object; rather, *the subject desires the object because the rival desires it.* In this triangular relationship it is the rival that is accorded the prominent role and serves as a model for the subject not only in regard to secondary matters as style and opinions but also, and more essentially, in regard to desires.[85]

There are a number of problems that Girard's theory raises. Chief amongst them is that it is not entirely clear what makes a model a model or a subject a subject in Girard's thought. It appears that he takes the relative position of each as a given of or, at least, engendered by the wider socio-cultural context. However, the relationship between the model and the subject and, even, the identity of the model and the subject is not static or fixed. Who is the model and who is the subject will shift depending upon the circumstances. Perhaps more importantly, the model is always under threat of losing his or her power, or the efficacy of that power, because he or she too has a *rival.* In this sense, the model is also intimately affected by the desires of another. Paradoxically, there may

[83] Ibid.

[84] Livingston P, *Models of Desire: René Girard and the Psychology of Mimesis* (Baltimore, 1992), 1.

[85] Emphasis in the original. Girard R, *Violence and the Sacred* (London, 1972), 11.

even be circumstances when the model and the subject are equal or that the differences between them, be they hierarchical or not, dissipate.

Further, the object in Girard's concept of mimetic desire appears to take on a subordinate role in generating rivalry. However, there may be instances where the object takes on a charismatic quality and the possession of an object by one party may have the capacity to confer, to paraphrase Girard, a greater plenitude of power on that party. This coalesces with Benjamin's idea that the aura of an object is generated by its control. Lastly, it would be wrong to see the subject as merely passive because his or her desires are mediated by another. Livingston argues that it is more useful to think of mediated desires as subject to selectivity.[86] This is an important point because the concept of selectivity assumes agency and provides an important rebuttal to the Darwinian notion that mimicry is a form of primitive automatism. There are defensive forms of mimesis as Caillois points out when discussing camouflage, but even here there is an important strategic element.[87]

The importance of Girard for my purposes is that he treats mimesis as a form of conflict. Girard argues that mimetic desire is a chief cause of societal conflict, which generates a structural crisis in society. The societal crisis (as he calls it) is caused by the erosion or the instability of the system of differences such as different identities, which underpins the social fabric.[88] The societal crisis is another way of referring to the paradox of the equalisation of the subject and the model or, even, the menace of mimicry taken to its logical conclusion. This loss of differences can take a number of forms, but all are ultimately mimetic. Girard calls them 'monstrous doublings'[89] (such as, the rivalry between two brothers) given the immensity of the societal crisis that they engender. The example of the rivalry between two brothers is particularly illuminating. Kluckhohn points out that the most common mythical conflict is the struggle between brothers.[90] This conflict is expressed in rivalry which leads to a cycle of violence. The rivalry of two brothers over the throne of their father King

[86] Livingstone, *Models of Desire*, above n.84, 20.

[87] Taussig, *Mimesis and Alterity*, above n.40, 64.

[88] Van der Walt J, *Law and Sacrifice: Towards a Post-Apartheid Theory of Law* (London, 2005), 227.

[89] Girard, *Violence and the Sacred*, above n.85, 64.

[90] Ibid.

is a classic archetypical example and, is an example which is not just mythical.[91] It has historical resonance. It is also an example which brings us closer to thinking about the contest over sovereign power.

In this respect the two brothers desire the same object — the throne.[92] At moments in the brothers' rivalry the differences between them dissipate. In this example of the two brothers who desire their father King's throne the system of differences in the given society (mythic or otherwise) would have a way in which entitlement to the throne is conferred on one of them (via custom or law). This could be by anointment of the father King before his death or conferral of title based on being the first born son. It is through rivalry that the system of differences blurs and the two become more equal. For example, the King may die before anointing a successor, reducing the differences between the brothers and sparking a crisis which may spiral into violence. The conflict between the two brothers will have a significant impact on the whole society such as segmentation or fragmentation of society via alliances and factions.[93] Girard argues that under such conditions society and culture becomes increasingly impossible.[94]

If the relationship between mimetic desire and violence is not channelled into ritual practices such as sacrifice, Girard argues that violence is pursued by the combatants in an absolute sense.[95] Violence becomes an end in itself like some ultimate prize. When violence becomes an end in itself we reach the peak of societal crisis and the peak of revenge and reprisal. As Girard puts it:

> There is never anything on one side of the system that cannot be found on the other side, provided we wait long enough. The quicker the rhythm of reprisals, the shorter the wait. The faster the blows rain down, the clearer it becomes that there is no difference between those who strike the blows and those who receive them. On both sides everything is equal; not only the desire, the violence, the strategy, but also the alternation of victory and defeat, of exaltation and despair.[96]

[91] Ibid, 64–65.
[92] Ibid, 66.
[93] Ibid, 67.
[94] Ibid, 66.
[95] Ibid, 160–161.
[96] Ibid, 163.

What is most striking is that the antagonists are truly doubles — captured in a mimetic relationship that is, essentially, reciprocal and equal.[97] Reciprocity here should not be thought of in terms of mutuality (which presupposes a different understanding of equality). Reciprocity is used in the sense that the fate of the two combatants are inextricably linked and captured within a "to and fro" conflict. However, there is a mutualness between the combatants which opens up towards mutuality or, at least, its possibility. We find that while identities seem to persist in conflict of this kind, the antagonists are in reality the same. The rivals see themselves as separated by formidable differences, but this is not the case as the conflict is founded upon the very reduction of differences or the engendering of radical equality. It seems, though Girard does not pursue this line of thought, that this radical equality is an opportunity to build different relationships between the combatants and assert new identities precisely because differences are undermined or even rendered unable to be convincingly reasserted. The closest approximation that we have to this idea is Benjamin's concept of 'divine violence', which is a power that destroys laws (as a structuring force in society) and is a precursor to a revolutionary form of violence.[98]

Girard's mimetic conflict is invariably in one way or another violent and incessantly threatens to descend into a full scale war. However, mimetic conflict need not be reduced merely to instances of violent conflict, particularly if we are to treat the desired object as a symbol, rather than a thing that can be possessed in a tangible, or perhaps more precisely, an empirical sense. The concept of mimetic conflict is capable of being applied to symbolic forms of conflict. Certainly, in the example of the spirit boat the object that the Shaman wishes to capture is the European, but the European as a symbol of power. Moreover, if we bring the discussion back to the object of the paper, Captain Cook has become part of the Australian Nation-State's stable of symbols, a symbol, as I have argued, the Rembarrnga contest (whether by claiming ownership of the symbol or by devaluing the Anglo-European version of Captain Cook). This is not to suggest that symbolic forms of conflict can be isolated from (for want of a better word) "real" conflict. As Harrison puts it:

[97] Ibid, 155.

[98] See Benjamin W, 'Critique of Violence' in Demetz P (ed), *Reflections: Essays, Aphorisms, Autobiographical Writings* (New York, 1978).

> Competition for power, wealth, prestige, legitimacy or other political resources seems always to be accompanied by a conflict over symbols, by struggles to control or manipulate such symbols in some vital way.[99]

However, symbolic conflict does not necessarily always engender 'real' conflict or violence.

Drawing on Bourdieu's concept of symbolic power, Harrison identifies four types of symbolic conflict: valuation contests, proprietary contests, innovation contests and expansionary contests. Valuation contests involve competitors ranking symbols according to some criterion of worth such as prestige, legitimacy or sacredness.[100] The aim of valuation contests is to raise the prestige and status of one group's symbols while at one and the same time devaluing another group's symbols.[101] Proprietary contests involve claims of proprietary rights over symbols and treat attempts by other groups to copy them as hostile acts.[102] The contestants or competitors agree on the prestige of the symbol but dispute the ownership.[103] Innovation contests are the creation of new symbols or the changing of old symbols,[104] whereas in expansionary contests the goal is to make the opposition adopt one's own symbols or to displace its competitor's symbols of identity with its own symbols.[105] To paraphrase Harrison, the resources for which the players are implicitly competing in an expansionary contest, seems to be people's political allegiances.[106] While these four types of symbolic conflict can be separated for analytical purposes, many conflicts will exhibit more than one type of symbolic contest.

Harrison calls symbolic conflict a 'zero sum game'.[107] The issue is not the quantity of the symbols (i.e., how many each group has) but the quality of the symbol or symbols. As Harrison puts it:

[99] Harrison S, 'Four Types of Symbolic Conflict', *The Journal of the Royal Anthropological Institute*, (1995) 1:2, 255.
[100] Ibid, 256.
[101] Ibid.
[102] Ibid, 258.
[103] Ibid, 259.
[104] Ibid, 261.
[105] Ibid, 263.
[106] Ibid, 265.
[107] Ibid, 269.

> In short, a characteristic of symbolic conflict is that it takes the form of a zero-sum game in which ratios and not quantities of symbolic capital are at issue, and in which any gain to one group or actor can only be made at the expense of some other or others.[108]

Harrison's zero sum game is similar to Frazer's magic of contact or contagion, discussed earlier, in that symbols when employed in symbolic conflict act on each other, either diminishing or increasing the value of the symbols held by a group.

Bourdieu argued that symbols represent the funds of 'symbolic capital'.[109] Symbolic capital is, as Harrison puts it, 'in part a disguised, mystified form of economic capital'.[110] Bourdieu developed his notion of symbolic capital by studying Kabyle society in Africa. The economic capital of the descent group in Kabyle society includes land, manpower and other material resources (its power in a tangible sense).[111] Its symbolic capital is its reputation or prestige and its economic capital can be added to or furthered by exploiting its symbolic capital.[112] However, I suggest that symbolic conflict can cause the same kind of societal crisis in Girard's terms. Symbolic contests between rivals (such as rivalry over the ownership of a symbol) can threaten to undermine the status of one group whose entitlement to economic capital is partly legitimised by its symbolic capital.

How then is societal crisis or the menace of mimicry resolved? Part of the answer lies in Girard's model of ritualised violence, which is the 'primitive' institution of sacrifice. My understanding of ritualised violence, like conflict, is a wider one, to include the western institution of law, but for the moment I will confine my comments to sacrifice. For Girard, sacrifice functions to prevent the type of reciprocal violence and conflict (putting aside symbolic conflict) discussed above. The sacrificial entity, and hence the word "scapegoat", 'is a substitute for all members of the community, offered up by members themselves.'[113] The sacrifice serves to protect the entire community from *its own* violence by choosing victims

[109] Bourdieu P, *The Logic of Practice* (Cambridge, 1990), 112-21.
[110] Harrison, 'Four Types of Symbolic Conflict', above n.99, 268.
[111] Ibid, 268.
[112] Ibid.
[113] Girard, *Violence and the Sacred*, above n.85, 8.

outside the community. Sacrifice is itself a form of violence — a very real violence — though one intended to prevent the eruption of reciprocal violence. Any dissension scattered throughout the community is drawn to the scapegoat and is eliminated by its sacrifice.[114] The sacrificial entity or victim can be actual or figurative, animate or inanimate but, most importantly as Girard puts it, 'always incapable of propagating further vengeance'.[115]

Sacrificial violence, as opposed to reciprocal violence, is the *unanimous* violence of the community. However, the dissension within the community is eliminated only on a temporary basis and requires repetition of the sacrificial process.[116] The institution of sacrifice replaces the vicious cycle of reciprocal violence with the vicious cycle of ritual violence which, unlike the destructive nature of reciprocal violence, is meant to be creative and protective in nature. Girard sees a difference between sacrifice (ritual violence) and law. The key difference between the two is that sacrifice *prevents* reciprocal violence, while the legal system in 'modern' societies *cures* reciprocal violence.[117]

There may be 'rudimentary' forms of curative institutions in some primitive societies, but according to Girard the establishment of the judiciary is the most efficient of all curative institutions.[118] Girard's theory echoes legal theories which seek to justify the centralised criminal justice system, in particular the theory of retributive justice in which the criminal justice system is seen to have rationalised the principle of vengeance. Girard argues that the efficiency of the modern legal system at curing reciprocal violence is founded in the recognition of the sovereignty and independence of the judiciary whose decisions, at least in principle, no group, not even the community as a whole can challenge.[119] As Johan van der Walt puts it discussing Girard, an independent judiciary 'terminates the cycle of revenge by staging its revenge as the revenge of everyone in society' and not on behalf of a faction or segment in society.[120]

[114] Ibid.
[115] Ibid.
[116] Ibid, 21.
[117] Ibid.
[118] Ibid, 21–22.
[119] Ibid, 23.
[120] Van der Walt, *Law and Sacrifice*, above n.88, 228.

It is at this juncture that I want to discuss Hobbes' theory of sovereign power, which is famously known as the Leviathan (that common power to keep the multitude in awe).[121] It is Hobbes' model of sovereignty that most graphically exemplifies the idea of a sovereign "body" that stages its revenge as the revenge of everyone in society. As Hobbes says:

> The only way to erect such a Common Power ... is, to conferre all their power and strength upon one Man, or upon one Assembly of men, that may reduce all their Wills, by plurality of voices, unto one Will: which is as much as to say to appoint one Man, or Assembly of men, to beare their Person; and every one to owne, and acknowledge himselfe to be Author of whatsoever he that so beareth their Person, shall Act, or cause to be Acted, in those things which concerne the Common Peace and Safetie; and therein to submit their Wills, every one to his Will, and their Judgments, to his Judgments ... This done, the multitude so united in one Person, is called a Commonwealth, in Latine Civitas. This is the generation of the great Leviathan or rather (to speak more reverently) of that *Mortall God,* to which we owe under the *Immortall God* our peace and defence.[122]

It might be objected that Hobbes' model of sovereignty is no longer adequate to capture the complex nature of sovereignty in the modern world. There has been a marked shift in legal and political theory away from thinking about sovereignty as exclusively belonging to one institution, be it a King, parliament or the modern executive, even if the institution in question is the State itself.[123] For example, Michel Foucault famously said that 'power is everywhere'; power having transformed in such a way that legal institutions can no longer be regarded as the locus of power.[124] More recently, there has been a 'questioning' of sovereignty in globalisation theory, because the nation-state (the nucleus of law or legal institutions) is seen as increasingly ineffectual in a globalising or

[121] Hobbes T, *Leviathan* (London, 1914), 88.

[122] Ibid, 89.

[123] See Veitch S, Christodoulidis E and Farmer L, *Jurisprudence: Themes and Concepts*, (London, 2007), 9–17.

[124] See Foucault M, *Discipline and Punish: The Birth of the Prison* (New York, 1977) and Foucault M, *History of Sexuality: Volume 1* (New York, 1990).

regionalising world.[125] Moreover, van der Walt criticises Girard's picture of the judiciary as representative of the community as untenable because the idea of a bounded cohesive community is equally untenable. However, there appears to be a marked schism between theory and the judicial decrees on sovereignty in the Australian colonial context. The "weakness" or "redundant" thesis of sovereignty, whatever its genesis, is simply indefensible in the wake of *Mabo* and the designation by the High Court of the Crown's sovereignty as non-justiceable. In the context of colonial relations in Australia sovereignty is unchallengeable.

I want then to suggest that Hobbes' model of sovereignty is mimetic, and in the next section I will discuss its persistence as a model of sovereignty in *Mabo*. There are three ways in which Hobbes' sovereignty is mimetic. First, Hobbes' infamous 'state of nature' bears a striking resemblance to Girard's societal crisis underscored by mimetic desire. Hobbes' state of nature is the state of 'Man' in 'his' natural condition. On one reading Hobbes' theory of the state of nature starts from a different premise to Girard's societal crisis. Hobbes does not start with a system of differences that erupts into societal crisis when those differences dissipate; Hobbes begins with a system of radical equality in which societal crisis is already embedded. 'Nature', Hobbes says, 'hath made men so equal, in faculties of body, and mind'.[126] Any differences in individual strength and intelligence is of little consequence to Hobbes, since they can be overcome in one way or another (such as, through alliances or 'secret machination').[127]

As in Girard's societal crisis Hobbes' radical equality generates rivalry. To quote Hobbes in full here:

> From this equality of ability, ariseth equality of hope in the attaining of our Ends. And therefore if any two men desire the same thing, which nevertheless they cannot both enjoy, they become enemies; and in the way to their End, (which is principally their owne conservation, and sometimes their delectation only), endeavour to destroy, or subdue one another.[128]

[125] See Veitch S, Christodoulidis E and Farmer L, *Jurisprudence: Themes and Concepts*, 53 and MacCormick N, *Questioning Sovereignty* (Oxford, 1999).

[126] Hobbes T, *Leviathan*, above n.121, 63.

[127] Ibid.

[128] Ibid.

It is not entirely clear whether Hobbes views desire as something generated by the appeal of the object or by rivalry. The motivations which underpin rivalry are as dynamic as Girard's. For example, Hobbes considers that there are three chief causes of 'quarrell': competition, diffidence and glory.[129] Competition concerns the invasion of another's person or property for 'gain'[130] (based presumably either on the convergence of desires on an object or even engendered by rivalry itself as seems indicated by Hobbes' use of 'delectation');[131] diffidence concerns the invasion of another's person or property for 'safety'[132] (diffidence could be thought of as a preventative strike caused by the anticipation of competition and rivalry and is, in this respect, a mediated motivation); and glory concerns the invasion of another's life or property for reputation.[133] Thus the state of nature, like societal crisis, is a state of conflict and contestation. Hobbes expresses the same concerns raised by Girard about the impossibility of society (no 'Industry', Science, 'Arts' and 'Letters') under such conditions.[134] This leads Hobbes to famously say: 'And the life of man, solitary, poore, nasty, brutish, and short'.[135]

Secondly, Hobbes' Leviathan has a preventative function as well as a curative one. Girard saw the legal system and the institution of sacrifice as having the same function — to deflect societal crisis — though the judiciary cures the societal crisis and sacrifice prevents it. It is curious that Girard maintains the distinction between sacrifice and law in this way. Although not expressly saying so, Girard appears to have seen unanimity as already existing in law, unlike the institution of sacrifice that creates or produces it, as one reason for maintaining this distinction. Hobbes no doubt would have agreed with Girard that unanimity exists in law. As representative of the multitude the Leviathan demands no less than that each individual renounce his or her particularity in so far as it conflicts with the 'Will' (or unanimity) of the community as embodied by the Leviathan. However, Girard did not consider that the judiciary also prevents the societal crisis and the erosion of differences that under-

[129] Ibid, 64.
[130] Ibid.
[131] Ibid, 63.
[132] Ibid, 64.
[133] Ibid.
[134] Ibid, 64–65.
[135] Ibid, 65.

scores the societal crisis by creating unanimity via the adjudicative function itself.

This creation of unanimity may in turn rest on the "sacrifice", in a symbolic sense, of an individual's or a group's particularity to the greater needs of the wider community (often thought of in terms of the "universal"). Van der Walt argues that a form of sacrifice inheres in adjudication when judges prefer one submission to other competing submissions and raise it to the status of legal precedent. As Van der Walt puts it:

> At issue with every judicial decision is the representation of the particular case, the inevitable representation that reduces to oneness the multiple conflicting desires and concerns that inform the law in a contradictory fashion. The sacrificial nature of the law stems from this need to reduce social ambiguity and the multiplicity that stems from it to simplicity and oneness.[136]

Law can, in some instances, inflict a symbolic violence by reducing individual or group experience to legal categories or by failing to accommodate particularity.

Furthermore, by accepting that unanimity was inbuilt in law Girard did not give his attention to the possibility of sovereignty (or law) becoming the object of contestation and becoming the trigger for societal crisis. For Hobbes, politically astute as he was, this was a possibility, indeed the possibility that he sought to render nugatory by erecting a sovereignty that was beyond contestation as well as by providing a graphic rationale for why sovereignty must be maintained. To bring the discussion back to the language of mimesis, the Leviathan cannot be copied or replicated as it has a unique presence in time and space as the creature of the social contract or the agreement between men to erect the Leviathan. However, once erected it was unique in another way. The Leviathan, as Hobbes puts it, is a 'mortall god',[137] imbued with the enduring qualities of a god, even though it is a human institution.

I want to suggest that Hobbes' state of nature is also preventative in providing a powerful motivation, which I call the "Leviathan Motivation", to protect "society" against the erosion of societal differences. This is an institutional motivation that I would like to later attribute to the High Court. For Hobbes, sovereignty is something that cannot be shared or

[136] Van der Walt, *Law and Sacrifice*, above n.88, 11.
[137] Hobbes, *Leviathan*, above n.121, 89.

divided, otherwise society is always under threat of descending into crisis, which he represents in the form of the state of nature. Hobbes presents us with — at least for him — another frightening mimetic image of a State in which sovereignty is either contested or divisible (Hobbes likely saw contestation and divisibility as the same thing as he refers to institutions based on a separation of powers as 'factional').[138]

> To what disease in the natural body of man I may exactly compare this irregularity of a Commonwealth, I know not. But I have seen a man that had another man growing out of his side, with a head, arms, breast, and stomach of his own: if he had had another man growing out of his other side, the comparison might then have been exact.[139]

Thirdly, Hobbes' Leviathan is itself mimetic. The mimetic nature of Hobbes' sovereign power is already evident in his example of what appears to be Siamese twins (twins are also an omen in some so-called primitive societies of a coming societal crisis). Hobbes was writing at a time when theology was no longer capable of legitimating sovereign power. It used to be that sovereign power in the person of the King or the prince mimicked the heavenly power of Christ or God. The King or the prince was God's representative on earth. James I of England was, in Buij's terms, one of the most outspoken rulers with respect to the divine appointment of the King,[140] but there is a long association of the Crown with God.[141] Hobbes raises Man to the status of God and the Leviathan appears to mimic Man. The Leviathan or State is, as Hobbes puts it, 'an Artificiall Man; though of greater stature and strength than the Naturall, for whose protection and defence it was intended; and in which, the *Sovereignty* is an Artificial *Soul*, as giving life and motion to the whole body'.[142]

I leave open the question whether western sovereignty is mimetic, separately to my contention that it is in the Australian colonial context. I

[138] Ibid, 176.

[139] Ibid.

[140] Buijs G, 'Que les Latins appellant maiestatem': An Exploration into the Theological Background of the Concept of Sovereignty' in Walker N, *Sovereignty in Transition* (Oxford, 2003), 231–232.

[141] See Kantorowicz, E H. *The King's Two Bodies: A Study in Medieval Political Theology* (New Jersey, 1957).

[142] Hobbes, *Leviathan*, above n.121, 9.

greatly doubt, in any event, that it can be isolated from the colonial context. As Anghie puts it, 'no adequate account of sovereignty can be given without analysing the constitutive effect of colonialism on sovereignty'.[143] Indeed, in the same breath that Hobbes doubts that the state of nature has existed all over the world, he notes 'there are places where they so live now'.[144] In Hobbes' words:

> For the savage people in many places of America, except the government of small families, the concord whereof dependeth on natural lust, have no government at all, and live at this day in that brutish manner, as I said before.[145]

At the same moment in time that sovereignty mimics Man, 'savage people' are set both outside and against sovereignty — simply put: a people without sovereignty. Hobbes' passage is reminiscent of Darwin's reaction to the Fuegians in so far as we are witness to the emergence of an original and by an original. Europeans are sovereign people and sovereignty is something that Europeans, not savage people, possess.

3. Contesting Sovereignty in Australia — Mimetic Strategies

3.1. The Contest over Sovereignty: *Mabo* and the Strategy of Native Title

I now turn to address the critical issue — what does it mean to say that sovereignty in Australia is mimetic? In this section I will turn my attention to *Mabo*, the High Court's recognition of native title. My starting point is not with native title, which I will return to below, it is with the contest over sovereignty in *Mabo*. It is not usual to discuss the case *as* a contest over sovereignty. In *Mabo*, we are told by the Court in no uncertain terms that sovereignty is not contestable and the literature on the subject has largely critically evaluated *Mabo* in these terms. Moreover, there was no contest over sovereignty in a purely technical sense. In legal

[143] Anghie A, 'Finding the Peripheries: Sovereignty and Colonialism in Nineteenth Century International Law', *Harvard International Law Journal*, (1999) 40:1, 1–80.

[144] Hobbes, *Leviathan*, above n.121, 64.

[145] Ibid.

terms sovereignty did not form part of the controversy which the Court was asked to decide. At the outset of Brennan J's judgment, considered the leading judgment in *Mabo*, the Crown's acquisition of sovereignty was said to have been conceded by the plaintiffs, Eddie Mabo and others, claiming on behalf of the Meriam people of the Murray Islands in the litigation. The question before the Court was the consequence of the Crown's acquisition of sovereignty over Australia and whether at the time of the assertion of sovereignty the Crown became the beneficial owner or proprietor of all lands.

If we enlarge the context of *Mabo* a different picture emerges. The judges were on notice from *Coe v Commonwealth* that Aboriginal claims of rights to land were wrapped up in assertions of Aboriginal sovereignty.[146] The lesser known *Coe v Commonwealth* was heard by the High Court in 1978, at the peak of an Aboriginal activism which had emerged in the early 1970s asserting pan-Aboriginal claims in the form of an Aboriginal nation, Aboriginal sovereignty and Aboriginal land rights. The most famous protest of that period was the establishment of the Aboriginal Tent Embassy in 1972 on the lawn of the then Parliament House in Canberra, in response to the refusal of the McMahon Coalition Government to recognise land rights. The rest of the '70s saw the cyclical police removal of the Aboriginal Tent Embassy and its reestablishment by the activists. In 1979, one year after *Coe v Commonwealth*, the Aboriginal Tent Embassy was re-established on top of Capital Hill, the ground of the proposed new Parliament House, as a 'National Aboriginal Government'.

The issue before the Court in *Coe v Commonwealth* was a seemingly benign one of whether leave should be granted to amend a statement of claim. But, the statement of claim included different formulations of claims to Aboriginal sovereignty as well as the contestation of the validity of the Crown's sovereignty in Australia, including the following claim: 'From time immemorial prior to 1770 the aboriginal nation had enjoyed exclusive sovereignty over the whole of the continent now called Australia'.[147] The statement of claim also included a raft of claims to Aboriginal proprietary rights to land in Australia. However, these claims and the sovereignty claims were largely intrinsically tied. For instance, the statement of claim states that Captain Cook wrongly proclaimed

[146] *Coe v Commonwealth* [1978] HCA 41.

'sovereignty *and* dominion' over the east coast of the continent; Captain Arthur Phillip wrongly claimed 'possession *and* occupation'; Captain Cook and Captain Arthur Phillip wrongly treated the continent as *terra nullius* (empty or waste land) 'whereas it was occupied by a sovereign aboriginal nation'; and as a nation, aboriginal people were entitled to 'the quiet enjoyment of their rights, privileges, interests, claims and entitlements in relation to lands' and were not to be dispossessed 'thereof' without 'bilateral treaty, lawful compensation and/or lawful international intervention'.[148]

Paragraph 23A of the statement of claim put the sovereignty claim in an unusual form:

> On November 2nd, 1976 members of the aboriginal nation including the Plaintiff planted their national flag on the beach at Dover, England, in the presence of witnesses and natives of the territory of the second named defendant and proclaimed sovereignty over all the territory of the second named Defendant, namely the United Kingdom of Great Britain and Northern Ireland. On the 9th day of April, 1977 the aboriginal nation confirmed this sovereignty over its lands, country and territory known as the Commonwealth of Australia by planting its flag in the presence of witnesses at Kurnell.[149]

Two out of four judges (Jacobs and Murphy JJ) thought that the claims to land could be separated out from the claims to sovereignty and would have granted leave to amend. The other two judges (Gibbs CJ and Aickin J) thought that the issues concerning land were arguable, but not in the form in which the land claim had been pleaded. The High Court had laid the foundations for a *Mabo* type decision; however, the ultimate result in *Coe v Commonwealth* was the striking out of the statement of claim and the burying of both the sovereignty and land claims.

In the years between *Coe v Commonwealth* and *Mabo* the form in which claims to sovereignty, pan-Aboriginal nationhood and government were made, started to look starkly more and more like that which it appears to be emulating. In 1990, two years before *Mabo* was heard, an 'Aboriginal Provisional Government' (APG) was set up by activists to

[148] Emphasis added. Ibid, [1].
[149] Ibid.

agitate for an Aboriginal 'State' with a federal type structure vesting power in Aboriginal communities to determine their own affairs and a national government with 'residual powers' to deal with foreign governments, coordinate 'some uniformity between Aboriginal communities' and so on.[150] The proposal also included the issuing of Aboriginal passports to put pressure on the Australian Government to recognise a separate Aboriginal nation/state.[151] The Aboriginal Government would operate alongside all other Governments, including the Australian Government, 'and not be subordinate to it'.[152] The context for the APG is the setting up of the Aboriginal and Torres Strait Islander Commission (ATSIC) two months beforehand. ATSIC was quasi-governmental (with a structure that mimicked the Federal Government) and acted as a service delivery provider to Aboriginal communities as well as a representative body. Its governing representatives were elected by Aboriginal people in periodic elections. However, it was funded and overseen by the Federal Government and thus remained subordinate to it.

Before moving on to *Mabo* I want to return to *Coe v Commonwealth* and to the incredulity with which the Court greeted the claim in paragraph 23A in the statement of claim, set out above. Chief Justice Gibbs was shocked that 'experienced counsel' who had appeared to argue the case before the Court strived to justify the statement of claim, 'including even paragraph 23A'.[153] Chief Justice Gibbs mentions paragraph 23A another two times calling it 'absurd' and 'vexatious' and 'no judge could in the proper exercise of his discretion permit the amendment of a pleading to put it in such a shape'.[154] Justice Jacobs said that it 'cannot be allowed' and doubted, unlike Gibbs CJ, that it was even 'seriously pressed'.[155] Justice Murphy said that the statement of claim 'exhibits a degree of irresponsibility rarely found in a statement intended to be seriously entertained by a court', noting as an example the claim on behalf of the

[150] Mansell M, 'Towards Aboriginal Sovereignty: Aboriginal Provisional Government' in Kerruish V, (ed) *Law, Laws and Aboriginal Peoples*, (Course Materials, Division of Law, Macquarie University, Sydney, 1997), 156 and 158.

[151] Ibid, 159.

[152] Ibid, 158.

[153] Gibbs CJ, *Coe v Commonwealth*, above n.146, [8].

[154] Gibbs CJ, ibid, [9] and [19].

[155] Jacobs J, ibid, [15].

aboriginal nation to the whole of the territory of the United Kingdom.[156] Perhaps not surprisingly the Court fails to tell us why it is such an absurd claim or something that surely can't have been seriously pressed, taking it as self-evident. The irony of the claim is lost on no one, and I suggest, it is certainly not lost on the judges.

This brings me to elucidating what I mean when I say that *Mabo* is a contest over sovereignty. *Mabo* needs to be read in the context of *Coe v Commonwealth* and Aboriginal claims to sovereignty and nationhood. The trigger of the contest from the Court's point of view, whether it grasps the contest in this way or not, is the menace of mimicry. The colonised assert, in various formulations, that they too are a sovereign people *equal* to the coloniser. And, it is that claim to equality that sparks a crisis for the Court. Chief amongst the "problems" for the Court is that the recognition of Aboriginal sovereignty would depreciate in one way or another, the nature of the Crown's sovereignty in Australia. For example, Aboriginal sovereignty clearly takes historical precedence over the Crown's. It might be objected that claims to sovereignty as a rightful property of Aboriginal people, especially a sovereignty that looks no different to the Crown's, actually underscores the authority of the Crown's sovereignty, making it paradoxically more authoritative and original. That is to say, Aboriginal sovereignty can be dismissed as a mere copying or, even, mockery rather than something that is intended to be taken seriously, invoking some of the issues discussed earlier in this paper in relation to Benjamin's discussion of art, aura and mechanical reproduction.

At first glance Aboriginal claims to sovereignty appear to underscore the power of the Crown's sovereignty. For example, Gibbs CJ in *Coe v Commonwealth* outrightly rejects any notion that there is an aboriginal nation, 'if by that expression is meant a people organised as a separate State or exercising any degree of sovereignty'.[157] Without the benefit of evidence Gibbs CJ states (or hopes) that 'they have no legislative, executive or judicial organs by which sovereignty might be exercised'.[158] However, there had been a significant levelling of differences between colonised and coloniser in the international setting that makes a response like Gibbs CJ's increasingly difficult to justify and sustain. By the time a

[156] Murphy J, ibid, [2].
[157] Gibbs CJ, ibid, [22].
[158] Gibbs CJ, ibid.

"competent" claim to indigenous rights finally reaches the Court in *Mabo*, the Court is deep within complicated mimetic territory.[159]

The ground had shifted from under the Court's feet with the post-colonial developments in the international setting, itself referable to the colonial struggles of the Other for independent "states", "nations", "sovereignty" or "self-determination". Numerous documents against discrimination based on race and protecting human rights had been published and exulted in the international sphere. Justifications of colonisation based on scientific racism or based on ethnocentrism, including *terra nullius*, had been denounced. In 1975 the International Court of Justice in the *Western Sahara Case* had declared that 'the concept of terra nullius, employed at all periods, to the brink of the twentieth century, to justify conquest and colonisation, stands condemned'.[160] *Terra nullius*, as Irene Watson puts it, became 'discredited as a tool for the colonisation and occupation of territories'.[161]

How then does the Court respond to the menace of mimicry? The rest of this chapter is devoted to answering this question. I argue that the Court responded with a mimetic strategy of its own.

The gardens were being tilled

The issue for decision in *Mabo* was whether the annexation of the Murray Islands to the State of Queensland vested an absolute form of ownership to all land in the Murray Islands, also known as beneficial ownership, in the Crown, thereby stripping the Meriam people 'of their right to occupy their ancestral lands'.[162] The Court accepted the assumption that Australia had been settled under the doctrine of *terra nullius*, which, in turn, underpinned the theory that the Crown became in law the sole proprietor of all lands in Australia. Therefore, it confined its 'critical examination' of the doctrine to the way in which indigenous rights and interests in land were made invisible by *terra nullius*.[163] *Terra nullius* in the literal sense of the term means unoccupied land. However, its significance in the colonial

[159] This is not to say that Court was not in complicated mimetic territory at the time of *Coe v Commonwealth*.

[160] Brennan J, *Mabo*, above n.3, [41].

[161] Watson, 'Buried Alive', above n.2, 257.

[162] Brennan J, *Mabo*, above n.3, [38].

[163] Brennan J, ibid, [28].

context was to consider inhabited land as 'practically unoccupied',[164] if the inhabitants were deemed 'low' on the 'scale of social organisation'.[165] Lord Sumner speaking for the Privy Council aptly sums up *terra nullius* in this way:

> The estimation of the rights of aboriginal tribes is always inherently difficult. Some tribes are so low in the scale of social organisation that their usage and conceptions of rights and duties are not to be reconciled with the institutions or legal ideas of civilised society. Such a gulf cannot be bridged. It would be idle to impute to such people some shadow of rights known to our law and then to transmute it into the substance of transferable rights of property as we know them.[166]

There had been a string of cases concerning Australia that supported this doctrine, including *Attorney General v Brown* where New South Wales was described as a wasteland with no proprietor other than the Crown.[167] In *Cooper v Stuart* the Privy Council described New South Wales as 'practically unoccupied, without settled inhabitants or settled law' at the time it was 'peacefully annexed' to the Crown.[168]

In re Southern Rhodesia indicates that the Privy Council were looking for — and not seeing — an aboriginality that it could reconcile with civilised society (as its measure of estimation). It is perhaps significant, seen in this light, that the Meriam people's 'gardening prowess' becomes the focal point of Brennan J's recognition of their relationship to land.[169] In the opening pages of his judgment Brennan J cites at length Moynihan J's description of the Meriam people at the end of the 18th century.

> The cultivated garden land was and is in the higher central portion of the island. There seems however in recent times a trend for cultivation to be in more close proximity with habitation. The groups of houses were and are organised in named villages.
>
> . . .

[164] From *Cooper v Stuart* cited in Brennan J, ibid, [36].
[165] Brennan J, ibid, [38].
[166] Cited by Brennan J, ibid.
[167] Brennan J, ibid, [25].
[168] Emphasis added. Brennan J, ibid, [36].
[169] Motha S, 'Mabo: Encountering the Epistemic Limit of the Recognition of "Difference"', above n.70, 81.

> Garden land is identified by reference to a named locality coupled with the name of relevant individuals if further differentiation is necessary. The Islands are not surveyed and boundaries are in terms of known land marks such as specific trees or mounds of rocks. Gardening was of the most profound importance to the inhabitants of Murray Island at and prior to European contact. Its importance seems to have transcended that of fishing. Gardening was important not only from the point of view of subsistence but to provide produce for consumption or exchange during the various rituals associated with different aspects of community life.[170]

There have been a number of celebratory, albeit critical, characterisations of *Mabo* as a recognition of 'difference' through law.[171] In contrast, Stewart Motha argues that the judgment really amounts to a recognition of 'sameness'.[172] If, as Motha suggests, we widen our context to take in the dominant 'Anglo-European conception' of relating to the land, the reason for Brennan J's focus on gardening becomes apparent.[173] As an example Motha cites John Locke's theory on the mixing of one's labour with the land as giving rise to legal rights of possession.[174]

> As much land as a man tills, plants, improves, cultivates, and can use the product of, so much is his property. He by his labour does, as it were, enclose it from the common.[175]

Further, the passage cited by Brennan J is overflowing with "signs" of a quintessential English connexion to land. From the organisation of huts into villages, the reference to boundaries (not quite fences but a type of enclosure nevertheless) and to the garden — all are historically English symbols of ownership, permanence, cultivation and improvement.[176]

Having found that the Meriam people have a relationship to the land that the Court can (literally) recognise, Brennan J comes to the view

[170] Moynihan J cited in Brennan J, *Mabo*, above n.3, [3].

[171] See Motha, 'Mabo: Encountering the Epistemic Limit of the Recognition of "Difference"', above n.70, 82.

[172] Ibid.

[173] Ibid.

[174] Ibid.

[175] Locke cited in ibid.

[176] Seed P, *Ceremonies of Possession in Europe's Conquest of the New World, 1492–1640* (Cambridge, 1995), 18–19 and 22.

that the most just course for the Court would be to overrule the existing authorities that disregarded the distinction between inhabited colonies that were *terra nullius* and those which were not. He thus imported the judgment into the mainland.[177] All indigenous inhabitants of Australia have proprietary interests in land or a native title capable of recognition by the common law, which is not extinguished on a 'mere change in sovereignty'.[178] Native title, Brennan J tells us, is not a creature of the common law. It has its origin in and is given its content by the traditional laws acknowledged by and the traditional customs observed by the indigenous inhabitants of a territory.[179] The nature and incidents of native title must be ascertained as a matter of fact by reference to those laws and customs.[180] The indigenous inhabitants of Australia are 'recast', in Motha's words, as 'proper(tied)' subjects.[181]

There is an extraordinary contradiction at work here. While the recognition of the Meriam people's relationship to land was based on 'sameness' we see a subtle shift to "similarities", invoking Bhabha's concept of the 'same, but not quite'. The garden was of central importance for the establishment of English colonies. Each European nation bidding for colonial expansion had ceremonies or rituals of possession intended to signify to each other universally clear acts of establishing colonies (though it is questionable how universally clear these acts of possession were even amongst European nations). The English, Seed argues, planted gardens and she notes the English preference to refer to its territories in the New World as 'plantations' rather than colonies.[182] Although Seed's examination of possession ceremonies is predominately concerned with the establishment of colonies in the New World, she nevertheless points out that gardening as a sign or, more precisely, act of possession continued well into the 18th century. Captain Cook was known to have planted gardens on some of the islands that he visited as one way of fulfilling orders from the Admiralty to take possession of settlements 'by Setting up Proper Marks

[177] Brennan J, *Mabo*, above n.3, [39].

[178] Brennan J, ibid, [61].

[179] Brennan J, ibid, [64].

[180] Brennan J, ibid.

[181] Motha S, 'Reconciliation as Domination' in S Veitch, *Law and the Politics of Reconciliation* (Hampshire, 2007), 74.

[182] Seed, *Ceremonies of Possession in Europe's Conquest of the New World, 1492-1640*, above n.176, 41.

and Inscriptions, as first discoverers and possessors'.[183] There is, in this respect, an important link between property and sovereignty and Seed's theory fits neatly within the linkage of property and sovereignty in European political thought. Locke, for example says, the 'great and chief end' of 'men uniting into commonwealths and putting themselves under government, is the preservation of their property'.[184]

This contradiction was likely at the back (or even forefront) of Brennan J's mind, when referring to the justification for the settlement of land under the doctrine of *terra nullius* as being that the land was uncultivated by the indigenous inhabitants, his Honour said:

> It may be doubted whether, even if these justifications were accepted, the facts would have sufficed to permit acquisition of the Murray Islands as though the Islands were terra nullius. The Meriam people were, as Moynihan J found, devoted gardeners.[185]

Do we not again see the menace of mimicry? Just as quickly as Brennan J opens up the possibility that the Crown's acquisition of sovereignty over the Murray Islands is invalid, his Honour immediately closes it down by saying that it is not something for the Court to 'canvass'.[186] This possibly also explains Brennan J's characterisation of Meriam society as regulated more by 'custom' than law.[187] It seems that indigenous people are not quite the same after all.

The mimetic strategy of sovereignty

This leads to my argument that there was a mimetic strategy of sovereignty in *Mabo*. I want to return to the central problematic here to tie the mimetic threads together. The essential point is this: the remarkable power of mimicry, hence its menacing nature, is that it depreciates the uniqueness of the "original". The Court, I contend, responds to this menace of mimicry with a double move of its own. On the one hand, the Court asserts sovereignty in a way that seeks to insulate it from mimicry and it does so by asserting that its sovereignty — in the form

[183] Ibid, 35–36.

[184] Locke J, 'Natural Rights and Civil Society' in Lessnoff M (ed), *Social Contract Theory* (Oxford, 1990), 101.

[185] Brennan J, *Mabo*, above n.3, [33].

[186] Brennan J, ibid.

[187] Brennan J, ibid, [3].

of the Crown — has the status of an original. In *Mabo*, we are witness once again to colonial "history" repeating itself as the Court behaves in a similar manner to Darwin by responding to "mimicry" with claims of Anglo-European originality. However, this claim to originality is infused with Hobbes' "paranoia" to erect a defensible Sovereign, so that the sovereignty that the Court deploys is authoritative in the Leviathan sense and is protected from mimetic contest. While on the other hand, underscoring Anglo-European claims to authoritativeness, the Court produces a "copy", or perhaps more precisely a "version" ("Aboriginal", "civilised savage", "gardener", "myth", "traditional laws and customs", "Crownless" and "native title") that is qualitatively different from the original ("Anglo-European", "civilised", "discoverer/possessor", "history", "sovereignty", "the Crown" and "tenure/property"). All copies or versions can be bundled into native title, but it should be kept in mind that there are a number of different levels of construction at work here. The original, which in *Mabo* is Anglo-European sovereignty, produces an impoverished copy of itself — native title — as a strategy and as an effect of its power *as* original.

But the Court's first rejoinder to the contest over sovereignty is a somewhat curious one, giving us an insight into its encounter with the societal crisis and the erosion of differences. This erosion of differences, put another way, is a crisis concerning its own foundations. We already see this crisis in Brennan J's questioning of the factual application of *terra nullius* to the Meriam Islands. The Court assesses its historical claim to sovereignty over Australia in the face of an Aboriginal challenge, but finds history wanting as a plausible reply.

Even though the Court informs us that sovereignty was not contested by the plaintiffs, it spends an unusual amount of time discussing the "question" of sovereignty. This questioning of sovereignty is accompanied by a hint of anxiety and, seemingly, confusion over what act constituted the Crown's acquisition of sovereignty over Australia. As a direct result there is some ambivalence about the legal significance of Captain Cook's act of possession as well as, interestingly, Captain Arthur Phillip's act of occupation. There were consistent references to 'our territory called New South Wales' in the Commissions from King George III to Captain Arthur Phillip, which indicated the view that the part of Australia that was annexed by Captain Cook, 'backed by an unexplored interior' of the colony, had already become British territory by virtue of 'discovery'.[188] However, Brennan J found such claims 'startling' and 'incredible', including under these terms a similar claim made by Isaacs J in *Williams v Attorney General for New South Wales* that when Governor Phillip received his first Commission from King George III, the whole of the lands of Australia 'were already in law the property of the King of England'.[189]

Justice Brennan considered that a sovereign could claim the territories newly discovered by their respective discoverers provided discovery was confirmed by occupation.[190] Justices Deane and Gaudron agreed with Brennan J that the preferable view is that the Crown established sovereignty on settlement of the colony.[191] Captain Arthur Phillip claimed possession for the Crown on arrival on 26 January 1788 and, curiously, once again claimed possession by reading out his second Commission on 7 February 1788 'with all due solemnity'.[192] Even on that approach, Deane and Gaudron JJ observed, 'there are problems about the establishment of the Colony in so far as the international law of the time is concerned'.[193] In Deane and Gaudron JJ's words:

> It is scarcely arguable that the establishment by Phillip in 1788 of the Penal Camp at Sydney Cove constituted occupation of the vast areas of the hinterland of eastern Australia designated by his Commissions.[194]

The same questioning of the reach of the Crown's jurisdiction on the establishment of the Penal Colony at Sydney could be asked again and again as inroads made across the continent by the British were similarly inchoate.[195] The Court in *Mabo* essentially scrutinises the historical acts that were challenged by the plaintiff in the statement of claim that was struck out in *Coe v Commonwealth* and with some consternation finds that these acts are riddled with problems. In the search for the original foundation of the Crown's sovereignty the Court confronts the critical

[188] Deane and Gaudron JJ, ibid, [3].
[189] Brennan J, ibid, [44].
[190] Brennan J, ibid, [33].
[191] Deane and Gaudron JJ, ibid, [3].
[192] Deane and Gaudron JJ, ibid.
[193] Deane and Gaudron JJ, ibid.
[194] Deane and Gaudron JJ cited in Motha, 'The Sovereign Event in a Nation's Law', *Law and Critique* 13:3 (2002), 319.
[195] See also Motha, ibid.

problem with colonial authority; it lacks, as Bhabha reminds us, authority of its own. Jacques Derrida makes a similar point, though he sees this as an inherent problem with sovereignty, in a sense its ontological deficiency (the 'origin of authority, the foundation or ground, the position of the law can't by definition rest on anything but themselves').[196]

How then does the Court assert its claim to sovereignty? The Court finds that sovereignty is embedded and entrenched *in* the concept of the Crown which, once the personal imprimatur of the Monarch or King, stands for the political and legal *authority* of institutions (such as, parliament, courts and the executive, or responsible government) in British and later Australian constitutionalism. The anxiousness and uncertainty with which the Court approaches the question of sovereignty suddenly falls away as the Court regains its confidence. 'We need not be concerned', Brennan J puts it, 'with the date on which sovereignty over Australian colonies was acquired'.[197] The power to extend its sovereignty and jurisdiction to a territory lay within the prerogative power of the Crown. In common law, prerogative powers are exceptional powers and privileges belonging exclusively to the Crown. Citing Diplock LJ, Brennan J states:

> It still lies within the prerogative power of the Crown to extend its sovereignty and jurisdiction to areas of land or sea over which it has not previously claimed or exercised sovereignty or jurisdiction.[198]

This assertion of sovereignty by the Crown is an 'act of state'. This act of state is non-justiciable, meaning it cannot be 'challenged, controlled or interfered with by the courts of that state'.[199]

It is intriguing that in the same year and in another constitutional context, a Court that shows a "modernising" willingness to supplant the Crown with the sovereignty of the Australian people in *Australian Capital Television Pty Limited v The Commonwealth*,[200] should invoke the Crown as the operative symbol of sovereignty in *Mabo*. International post-colonial and separatist struggles illustrated the incredible power of the symbol of a

[196] Derrida J, 'Force of Law: The "Mystical Foundation of Sovereignty"', (1990) 11 *Card. L. Rev.*, 943.

[197] Brennan J, *Mabo*, above n.3, [44].

[198] Brennan J, ibid, [31].

[199] Gibbs CJ cited in Brennan J, ibid, [31].

[200] (1992) 177 CLR 106.

"sovereign people" to successfully underpin claims to political autonomy, as the characterisation of one people as sovereign and another people as not sovereign could not be sustained without applying some kind of discriminatory discourse. However, the Court refuses to meet the Aboriginal challenge to also be a sovereign people on an equal playing field. I say "operative symbol" here because the Crown as it is used by the Court clearly stands for sovereignty and this rendering produces two complementary effects. First, the Court invokes a symbol that is a product of Anglo-European history, occupying a unique place in space and time. This uniqueness of the Crown is generated, as Benjamin would say, by its ownership and control. Simply put: the Crown belongs exclusively to the Anglo-European constitutional tradition and, as such, so does sovereignty. Secondly, the Crown is used as "indicia" to deny the existence of an Aboriginal sovereignty. We see this explicitly in Gibbs CJ's assertion in *Coe v Commonwealth*, as if a page out of Hobbes' *Leviathan*, that Aboriginal people have no organs by which sovereignty might be exercised, but the *Mabo* judges, acknowledging the era of post-colonial "equality", are much more circumspect.

Like Darwin's claim to originality, the Court's own claim through the symbol of the Crown relies upon the existence of an Other. The shortcoming of Derrida's analysis that sovereignty rests upon itself or upon its own assertion is that it fails to see sovereignty as something that is contextual or relational. The Court's assertion of sovereignty is not made in a vacuum, though it is clear that the Court thinks that this is the case, it is made in the colonial context. The Court's claim that sovereignty is an exclusive Anglo-European possession, like the magic of contagion or the zero-sum game, is intimately connected to the denial of Aboriginal sovereignty. To complete Bhabha's claim: colonial authority only gains authority or, as I put it, "authoritativeness" belatedly in the colonial context and this authoritativeness is built on the back of the production of discriminatory differences. In the Australian context the discriminatory difference is *terra nullius*, which, as I indicated above, could no longer underwrite colonisation in the new international climate that the Court finds itself in. 'Whatever the justification advanced in earlier days', Brennan J acknowledges, 'an unjust and discriminatory doctrine of that kind can no longer be accepted'.[201] However, while the Court seeks a reformed

[201] Brennan J, *Mabo*, above n.3, [42].

and recognised proper(tied) subject, its reformation and recognition is limited. Faced with all the consequences — societal, legal and political — that flow from the recognition of radical equality, the Court traverses a predictable path. It refuses to upset the structural differences (such as, ownership of property) that underscores the wider society; that is, what Brennan J calls in highly abstract terms the 'skeletal principle'. Law is a key structuring force that reinforces wider societal or structural differences and it cannot, as the Court tells us, be 'destroyed' or 'fractured'. The 'peace and order of Australian society', as Brennan J puts it, 'is built on the legal system'.[202]

It is through the "recognition" of native title that the Court rejects the entirety of the Aboriginal claim to equality — land rights *and* sovereignty — by creating native title, an imperfect rendering of Anglo-European sovereignty (and property). The impoverishment of native title is caused by its severance from an Aboriginal sovereignty or sovereignties equal to Anglo-European sovereignty or, even, an inkling of sovereignty able to withstand the full force colonisation, like the domestic dependent nation status accorded to Native American Indians. Whether this limited recognition and reformation is a conscious move is not important, it is both a strategy and effect of the model of sovereignty that the Court deploys in *Mabo*: the Leviathan, albeit in monarchical clothing.

Sovereignty in *Mabo* is overflowing with the Leviathan, which by its nature is a sovereign power that prevents contestation by demanding, backed by threat of force, political allegiance of all those within its jurisdiction. There is an assimilative project at work in *Mabo*. In Harrison's terms, this is an expansionary project where symbols are imposed on another group as a strategy of political allegiance (allegiance and subjecthood are two sides of the same coin in British constitutional thought). Aboriginal people are brought within the scope of the power of the object of contestation. On the acquisition by the Crown of sovereignty in Australia, the indigenous inhabitants automatically became subjects of the British Crown; the multiplicity of Aboriginal voices became one voice, subsumed into the unanimity of the Leviathan. As Justice Brennan puts it:

[202] Brennan J, ibid, [29].

> Thus the Meriam people in 1879, like Australian Aborigines in earlier times, *became British subjects owing allegiance to the Imperial Sovereign* entitled to such rights and privileges and subject to such liabilities as the common law and applicable statutes provided.[203]

However, the Court's assimilative project is incomplete or riddled with ambivalence. The indigenous inhabitants of Australia are, as already intimated in the Court's discussion of the Meriam peoples, proper(tied) subjects with a difference. Native title is not to be regarded, the Court tells us, as equivalent to the doctrine of tenure (the technical legal term for Anglo-European property) because it does not owe its existence to the Crown but to the traditional laws and customs of Aboriginal people.

In the construction of Anglo-European sovereignty we see sovereignty and property firmly wrapped up together in what the Court calls the Crown's 'radical title'. The link between property and the Crown is a fundamental one, much like the link between "improvements" to land and sovereignty discussed above. On the acquisition of sovereignty the Crown acquires radical title to the various colonial territories making up Australia. The Crown does not become the owner of all lands in Australia when it acquired sovereignty, but holds the ultimate right to exercise power in respect to land within its territory. Radical title, a relic from the English feudal system of landholding, is a 'postulate to support the exercise of sovereign power' in relation to territory.[204] In Brennan J's words:

> It is a postulate of the doctrine of tenure and a concomitant of sovereignty. As a sovereign enjoys supreme legal authority in and over a territory, the sovereign has power to prescribe what parcels of land and what interests in those parcels should be enjoyed by others and what parcels of land should be kept as the sovereign's beneficial demesne.[205]

We can hear the echoes of Locke's claim that the chief end of government is to preserve property but, it seems, not native title. Native title can be easily extinguished by an exercise of the paramount power or radical title of the Crown and, in this respect, lacks the constitutional, legal,

[203] Emphasis added. Brennan J, ibid, [36].
[204] Brennan J, ibid, [56].
[205] Brennan J, ibid, [50].

moral and political protections accorded, as almost a matter of obsession in Anglo-European thought, to Anglo-European private property. Even as British "subjects" Aboriginal peoples are impoverished. Since Anglo-European settlement of Australia, 'many clans or groups of indigenous people have been physically separated from their traditional land and have lost their connexion with it'.[206] They were 'dispossessed', Brennan J says, 'by the Crown's exercise of its sovereign powers to grant land to whom it chose and to appropriate to itself the beneficial ownership of parcels of land for the Crown's purposes'.[207]

In contrast to the awesome power of the Crown's sovereignty, Aboriginal laws and customs are apparently 'fickle'[208] and able to be lost. There is no acknowledgment by Brennan J of an Aboriginal form of sovereignty and one can only assume as I have already suggested, that the dominant, although not absolute,[209] historical bias that Aboriginal people have no political organisation or a 'fragile'[210] one remains at large in *Mabo*. Justice Brennan's rendering of the Meriam people's proprietary interest in land as customary rather than guaranteed by law is indicative of this thinking. This bias is perfected by the High Court ten years later in *Members of the Yorta Yorta Aboriginal Community v Victoria*.[211] Chief Justice Gleeson, Gummow and Hayne JJ say:

> But what the assertion of sovereignty by the British Crown necessarily entailed was that there could thereafter be no parallel law-making system in the territory over which it asserted sovereignty. To hold otherwise would be to deny the acquisition of sovereignty and as has been pointed out earlier, that is not permissible.[212]

[206] Brennan J, ibid, [66].

[207] Brennan J, ibid, [82].

[208] Kerruish V and Perrin C, 'Awash in Colonialism', *Alternative Law Journal*, 24:1 (1999), 4.

[209] See Jacobs J and Murphy J, *Coe v Commonwealth*, above n.146, [12] and [4]. Both where willing to hear a contest over whether Australia was annexed to the Crown by means of settlement or by conquest. The latter would have recognised a lesser form of sovereignty in Aboriginal people.

[210] Kerruish V and Perrin C, 'Awash in Colonialism', 4.

[211] [2002] HCA 58 (hereafter *Yorta Yorta*).

[212] Gleeson CJ, Gummow and Hayne JJ, ibid, [44].

Sovereignty in Australia in the Wake of Captain Cook

In this paper I have sought to show that sovereignty in Australia is mimetic. Colonial power and authority in Australia depends upon the production of discriminatory identities to avert conflict over land between Aboriginal people and Anglo-European Australians (used here interchangeably with the term "nation" that the High Court chose to employ in *Mabo*). Aboriginal dispossession, Brennan J acknowledges, 'underwrote the development of the nation'[213] and in so far as land has been alienated by a valid Crown grant there can be no contest over that grant of land.

However, *Mabo* was heralded by the Court as a 'retreat from injustice',[214] a retreat from discrimination and the ushering in of equality before the law, all aspirations of the contemporary Australian legal system.[215] As Brennan J put it, to maintain the authority of the cases on *terra nullius* 'would destroy the equality of all Australian citizens before the law'.[216] Nevertheless, the recognition of the Aboriginal claim to equality was incomplete and this, I suggest, was partly due to the Court's fear of unleashing the mimetic contest into and onto the 'nation'. One might wonder whether Brennan J would have been so eager to find that native title survives the assertion of the Crown's sovereignty, if he did not believe that its continued existence was 'exceptional'.[217]

> Of course, since European settlement of Australia, many clans or groups of indigenous people have been physically separated from their traditional land and have lost their connexion with it. But that is not the universal position. It is clearly not the position of the Meriam people.[218]

The controversy generated over *Wik Peoples v State of Queensland*[219] a mere four years after *Mabo* is especially telling. A divided Court in *Wik* (Brennan CJ was one of the dissenting judges) expanded native title to include properties covered by a Crown grant of a pastoral lease, exposing a larger portion of land in Australia to native title. The decision unleashed

[213] Brennan J, *Mabo*, above n.3, [82].
[214] Kerruish and Perrin, 'Awash in Colonialism', above n.208, 4
[215] Brennan J, *Mabo*, [29]
[216] Brennan J, ibid, [63].
[217] Kerruish and Perrin, 'Awash in Colonialism', 4.
[218] Brennan J, *Mabo*, above n.3, [66].
[219] (1996) 187 CLR 1 (hereafter *Wik*).

what can only be described as mass national hysteria that the average person was under threat of losing his or her "backyard" to native title claims. This hysteria gives us a real insight into the existence of a fear deeply embedded in the national consciousness that tenure or private property, the bedrock of the Anglo-European claim to land in Australia, holds the pulse of the "original" Aboriginal proprietary interest.[220]

What was missed by the 'nation' was that the majority judges in *Wik* had recognised, though it is unlikely that this recognition was conscious, the aspect of reciprocity fundamental to mimetic conflict. The desire of Aboriginal people and Anglo-European Australians to have their interest in land recognised and secured, however that interest is manifested, is fundamentally the same. Instead of seeing the object of the interest — property — as exclusively belonging to Anglo-European interests the Court considered that both interests, Aboriginal and Anglo-European, could coexist in the same property. The fleeting promise of *Wik* was the concept of the co-existence of different forms of land uses and interests, though with one caveat, so to speak. When the two uses came into conflict, native title, still the impoverished copy, would give way to Anglo-European property rights. However, the symbolism of co-existence contained the seeds of a more significant retreat from colonial injustice. The post-*Wik* calls of the Howard Federal Government for "certainty" of effectively the exclusivity of Anglo-European ownership, later turning into a legislative response of extinguishment of native title, signalled once again the Leviathan motivation towards coercion rather than contestation. The opportunity was lost to grasp that even though contestation always threatens to descend into further crisis, it generates reciprocity, which is a step towards mutuality and a more profound basis for a progressive dialogue.

What remains to ask is what is the status of Captain Cook in the wake of *Mabo*? This question becomes more pertinent since *Wik*. In *Mabo*'s wake native title has started to work injustices in the form of extinguishment (*Wik* in the context of High Court native title cases is itself exceptional). Motha points out that the Court's assertion of sovereignty in *Mabo* paradoxically repeats and retreats from the original foundation of Australian law.[221] Captain Cook's place and role within the pantheon of sovereign acts remains intact. As I suggested in the first section of this

[220] Many thanks to Dean Wadiwell for this insight.
[221] Motha, 'The Sovereign Event in a Nation's Law', above n.194, 317.

paper, his act of most consequence was to bring the continent within the framework of Anglo-European "history". In law he brings the continent within the framework of the Crown's prerogative which, indeed, works an extraordinary power, in fact, a sovereign power in *Mabo* and beyond. The presence of Captain Cook's act of bringing Australia within the fold of Anglo-European history and sovereignty (the two intrinsically linked) is conveyed in Brennan J's 'tide of history' metaphor in *Mabo*. As Brennan J puts it:

> When the tide of history has washed away any real acknowledgement of traditional law and any real observance of traditional customs, the foundation of native title has ceased.[222]

As Watson frames it, *Mabo* 'legitimised Cook's violent arrival'.[223] The 'tide of history' metaphor was invoked by Olney J of the Federal Court in *Members of the Yorta Yorta Aboriginal Community v Victoria*[224] to justify his decision that the descendents of the Yorta Yorta peoples had ceased to occupy their traditional lands in accordance with their traditional laws and customs. His honour effectively found that the tide of history, presumably itself a further metaphor for the march the civilisation, had stripped the Yorta Yorta peoples of their nativeness. This is the realisation of the assimilative project of western mimetic sovereignty where the "native" loses his or her indigenous claim — the original claim — to land because they become too much like "us" '.

Too Many Captain Cooks grasps the 'tide of history' all too well when at the critical juncture in the Rembarrnga dreaming the power of Captain Cook the ancestor is replicated in the Captain Cooks that come later to Australian shores. For the Rembarrnga, the (sovereign) power of Captain Cook and his sons are equivalent (the Rembarrnga are a sovereign people), but the balance between the two is ruptured by the multiplication of the sons of Captain Cook — there are just too many of them. The brilliance of *Too Many Captain Cooks* is that it provides us with a haunting picture of the colonial project as a mimetic one while, at one and the same time, showing us that there is some scope for resistance in the very symbols that are used by colonial power and authority to dominate.

[222] Brennan J, *Mabo*, above n.3, [66].
[223] Watson, 'Buried Alive', above n.2, 264.
[224] [1998] FCA 1606.

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