Dialectical Logic and Indiscrete Models[1]

Charlie Donahue[*]

These laws [of logic] are the work of thought itself, and not a fact which [thought] finds and must submit to.[2]

G. W. F. Hegel 1830

Why are the objects that we want to take into account in dialectical logic not severally independent [isolated] things? What is there to relate entities internally as distinct from externally?[3]

Kerruish and Petersen

[With classical logic] the objective state of affairs is determined before we start reasoning.[4]

Kerruish and Petersen

It might be argued that if α is a state, then all the information that is relevant to possible transitions is contained in α alone.[5]

J. M. Dunn

Uwe Petersen[6] offers a unique perspective on the philosophical significance of substructural logics.[7] He argues that such logics encourage us to revisit Hegelian dialectical theses. The dialectical features that interest Petersen reside in proof theory. I note that these proof-theoretic characteristics have correlates in a certain class of models for non-classical logics.

Ternary operations on indices play a prominent role in the models for many deviant logics. Michael Dunn embraces an uncivilized interpretation[8] of such operations — the resulting monsters provide our correlates.[9]

[*]University of Massachusetts at Dartmouth, `cdonahue@umassd.edu`. Notes are at the end.

1. Some basic themes of dialectical proof theory

Petersen has written extensively on dialectical logic. He focuses on some flaws in classical thought.

- Classical thought assumes that objects[10] are severally isolated and independent

Which objects? That will soon be clarified in two ways: proof-theoretically and model-theoretically.[11] Proof-theoretically these turn out to be something like 'bundles of information'; model-theoretically — indices in a model. This classical prejudice towards only such isolated, independent objects — manifests itself in a restriction to only external relations.[12] The cure requires the recognition of internal relations[13] between objects.

- Classical thought misses the inescapable double character of conceptual thought — concealing a systematic ambiguity[14]

Gödel numbers provide an example of this 'duality of function'. One and the same formal entity can play two roles — functioning as "a innocent number" and (simultaneously) as "a hieroglyphic behind which a complex proposition is hiding".[15] The point could as well be made with certain classical conundrums — for example that arising from Frege's conception of a property.[16] Ternary relations on indices present the possibility for the correlated — model-theoretic duality of function — such that one index may play two roles simultaneously.[17]

- Classical logicians deal with wffs as types — not as tokens.

And:

- Classically — proofs are thought of as objects — not as acts.

Proof-theoretically the structural rule contraction (among other things) guarantees that formulas will be taken as types and that proofs will be taken as objects. Dropping contraction will allow formulas to be taken as tokens and proofs to be taken as acts. Model-theoretically — reasoning transcendent truth (RTT) is the assumption that whatever adjudicates our semantic assignments is fixed before any semantic analysis can commence. David Kaplan's rejection of 'monsters' and Graham Priest's requirement of 'explanatory grunt' in any semantics for ternary operators display fealty to RTT.

1.1. How could internal relatedness be manifested proof-theoretically?

The answer comes from the substructural analysis of logic. Gentzen's sequent calculus substructural logics[18] allows for the *explicit recognition* of the role of structural rules (that recognize various ways of bundling premises) along with operational rules for the logical constants.

Neglect of premise combination[19] amounts to the *unstated* assumption that premise combination always follows set union — which is associative, commutative, extensional and idempotent. That in turn builds in structural principles such as associativity, commutation, weakening[20] and contraction. And that in turn restricts the sorts of information expressible in our logic. The deduction theorem transfers the properties assumed for premise bunching to the conditional. Proof-theoretically internal relatedness arises once we drop the structural principle called 'contraction'.

We will find that internal relatedness is expressed model-theoretically when the indices (worlds) are no longer required to be discrete[21] — as is required with any robust ternary operator.

1.2. Why be contraction free?

One particularly interesting[22] feature of the substructural approach is that the very same rules can be used for the operators in logics which are classical, intuitionist, relevant, linear and so on. This is remarkable because combatants on both sides of philosophical arguments — such as over realism — have standardly agreed that the rubber meets the road in adjudicating *which* are the *correct* definitions of the logical constants.

It turns out that there is a deeper issue than operational definitions — What are we willing to countenance as information? More specifically: How are the premises to be bunched?[23] And that, in turn, is determined by which differences in structural rules are condoned.[24] Those different assumptions about the possible structure of information reflect differences over 'how inference in the abstract is to be structured'.[25]

As an example consider the relevant objection to classical logic. In the end it has to do with whether on not we adopt weakening.[26] We were all wrong to assume that differences over the operational definitions of the logical operators are required to individuate logics.

One structural rule plays an especially important role with respect to a dialectical revision — contraction: (according to which) if a conclusion

follows from two assumptions of the same thing then it follows from just one. Linear logic is distinguished in that it drops contraction (along with weakening). In linear logic — formulae are taken as *concrete resources* whose multiplicity has to be taken into account.[27]

Here are some[28] sorts of reasons in favor of abandoning contraction:

- The absence of contraction is a safeguard against the property theoretic paradoxes.[29]
- The logic of information flow clearly requires relinquishing contraction.[30]
- Jacques Dubucs and Mathieu Marion[31] offer a new anti-Platonism that eschews Michael Dummett's (in principle) effective assertability conditions for conditions that are feasible in practice. This feasibility cannot be expressed by any Platonist conception of proof. In particular their feasibility requires the rejection of contraction.
- Resource consciousness: contraction free systems can display the 'cost' of running programs.

What is most relevant to the present investigation is that if contraction is rejected we can make sense of the transition to thinking of wffs as tokens (not types) and to proofs as acts (not objects).[32]

> By constraining discharge of hypotheses to be exactly up to one occurrence, namely to be both mandatory and unique, one takes up logic towards a fine-grained taxonomy of the activity of justification; justification is now analyzed act by act, and no longer by reference to undifferentiated types of acts.[33]
>
> Jacques Dubucs

2. Model-theoretic correlates

Let's preview the correlations between dialectic proof and model theory:

Proof theory rejects the assumption that objects are severally isolated and independent.	⇒	The indices of models need not be discrete.[34]

The isolation is repaired by recognizing an 'internal relatedness' between bits of information.	⇒	We drop the assumption that the information[35] at a given state (world) is (by itself) sufficient to express all possible transitions.[36]
We reject contraction which allows for reading wffs as tokens and proofs as acts.	⇒	The internal relatedness that corrects the classical isolation can be expressed by a robust ternary operation (which appreciates indices as tokens).
Conceptual thought has an inescapable double character — concealing a systematic ambiguity.	⇒	An inevitable duality of information (data) and computation (programs) violates types restrictions.

One way to present this is to apply Dunn's insights to a certain interpretation of modal logics with 'actually' operators. First we need a few words about ternary operators for relevant and two-dimensional logics.[37]

3. Ternary operators

> It does not matter what takes the place of the seeing-relation among players, so long as it is some kind of dyadic relation.[38]
>
> G. E. Hughes and Max Cresswell

Indexed logics[39] provide for the evaluation of any given wff at distinct indices or points. The classic example is, obviously, the standard modal logics — which allow wffs to be evaluated at different worlds. A number of assumptions were built into the original Kripke-style modal logics. Worlds were assumed to be consistent and complete. Relevant logicians like Richard Routley correctly railed against the expressive limitation of this. But there were more subtle assumptions. The worlds in the classical systems were assumed to be discrete. The full appreciation of what constitutes discretion with respect to indices will emerge.

3.1. Relevant B logics

It is obvious that anyone interested in relevance will not want $q \rightarrow (p \rightarrow p)$ to be valid. If our semantics is restricted to only binary relations we are in trouble. Why?

A binary relation would inevitably introduce irrelevance because since $p \rightarrow p$ is true in all worlds $q \rightarrow (p \rightarrow p)$ would be valid.

The whole motivation for the relevant inference tickets is to liberate the conditional[40] from the narrow paradigm according to which it requires an extensional reduction. For the first time information about one (formal) situation can give us information about another situation. That is indiscrete.

The system B gets *more discriminatory ability* by defining the conditional via the ternary relation — allowing the exploitation of the conditional at non-normal worlds under specified conditions. Normality conditions are stipulated which guarantee that the base (normal) world does not fully exploit the ternary relation. Basically what we are doing is allowing for a range of conditions to apply to the non-normal worlds while quarantining their effect such that they do not infect the normal world. The general effect of restrictions placed on the accessibility relation R in B logics is to block potential counter-examples in the ternary framework — allowing more valid forms to blossom.

The interpretation of such ternary relations is an extremely divisive question — and an extremely important one.

3.2. Discretion is violated by the inclusion of robust ternary operators

This is how discretion is violated by the inclusion of (robust) ternary relations.

Any ternary relation challenges[41] the possibility of restricting our semantics to *just* external[42] relations between worlds. Why?

Given just worlds 1 and 2 (at a time) — they can relate to each other (or not) but given 1, 2 *and* 3 — a new possibility opens up. One of these, in addition to being a discrete individual, can index sets of ordered pairs thus introducing something *beyond* an external discrete relation.[43]

This is where the distinction between robust and anemic ternary operators comes in. Anemic relations are those which are constrained just so that they cannot violate discretion. The cost is in expressive power.[44]

Dunn's interpretation of ternary relations is robust. We introduce 'actually' enriched modal languages first. The reason is that the application of Dunn's interpretation of ternary relations is particularly interesting when applied to the ternary operators that most naturally express (robust) two-dimensional logics. See appendix I for some of Dunn's own words on ternary relations. I will return to his insights below. First an 'actual' argument.

4. How to 'actually' avoid being too-much-of-this-world

The key problem for any modal logic with an actuality operator[45] is how to get generalizations of such principles without submitting to ethnocentrism — being too-much-of-this-world.

David Lewis offers a rebuttal to a possible rejoinder by Saint Anselm to one of his objections to the Ontological Argument. We generalize his argument to any instance of actualizing a modal principle.

    i. You have a metaphysical principle that you propose to *actualize*.[46]
    ii. With the *wide scoped* reading of the actuality operator the principle loses its status as a necessary truth — but it is required to keep that modal status if it is to secure its metaphysical pedigree.
    iii. That is, if the actuality operator's influence does not transcend the wide scope necessity operator it cannot solve the puzzle. But it is impotent if narrowly scoped.
    iv. It appears that the *only* generalizations possible either do nothing or get you back to ethnocentrism.[47]
    v. Thus actualization must be rejected.

Lewis' objection *does* indicate[48] why a one-dimensional analysis of 'actually' cannot work. Scope can't cope.[49]

But the conclusion — which is not explicitly stated in full generality[50] — is clearly too strong — it requires that we can *never* generalize on any actualized philosophical principles.[51] That is an extremely heavy conclusion. Lewis simply assumes that he has considered *all* possible generalizations. This is false — *unless* we assume that certain very general features[52] of classical model theory cannot be up for revision.

Lewis does not state premise (iv.) but without it his argument does not go through. According to (iv.) that there can be *no* generalization

that does the trick — transcending the power of the necessity operator (required to solve the puzzle) *without* losing modal generality.

This amounts to blocking the use of actualized modal principles from *ever* having general philosophical application. That ought to draw extreme skepticism from philosophers.[53] But it did not since this analysis is nourished by a more profound (deeply concealed) prejudice.

David Lewis later introduces a sort of pluralism that avoids this criticism. That pluralism is open to different criticism due to an unwarranted reliance on realist assumptions.[54] That argument is beyond the scope of this article. For now I simply claim that the proper generalization of 'actually' in modal contexts is achieved by adopting a two-dimensional logic.

There are (at least) two kinds of two-dimensional logic — anemic and robust. The latter emerges from Dunn's analysis of ternary relations.[55] That will be our correlate to the proof-theoretic dialectical theses above.

5. The two-dimensional fix

A new two-dimensional modal operator is required. It reads 'at *any* world j — with j taken as actual'.

That new operator is best represented via a ternary relation. It adds to the binary relations (used in standard Kripke modal logics to express *external* relations between worlds) a new point or world (for the perspective). As noted above the formal move to a ternary relation opens up the possibility to go beyond external discrete relations between worlds. A robust ternary operator exploits this possibility — an anemic one invokes a restriction forbidding that expressive power.

The third point has an inescapable *duality of function* because it picks out a certain world (as does any point) but *at the same time* it functions as the initiator of sets of worlds as actualized by itself. It does this by indexing sets of ordered pairs. This clearly violates types restrictions. Dunn will celebrate this.[56]

A sufficient condition of distinguishing between anemic and robust ternary relations is the violation of Åqvist's constraint[57] — according to which: If any two points differ *only* with respect to their perspectival coordinate then they must agree on their assignments to atomics.

Kaplan's insistence on avoiding monsters is an instance of an abiding faith in Åqvist's constraint.

> The liberating effect of giving up the classical faith ... is immense.[58]
>
> Richard Routley

6. Applying Dunn's insights on robust ternary operator to 'actual' logics (beyond civilized functions)

Dunn's analysis can be applied to a ternary relation for an 'actual' two-dimensional logic. The new index in our ordered triples is for the perspective world — 'the world taken as actual' at some point in an interpretation. All worlds get a shot at that role — thus ethnocentrism is avoided. But for any particular such triple the world in that new perspective position is indiscrete in that it enjoys a special second role (besides the one all indices fulfill of representing some particular situation) — that of *indexing* the following ordered pair of (standard) worlds. This duality of function is ineliminable if we are to have a robust ternary operator with maximum expressive power. This clearly amounts to indiscretion — a non-external relation — and a violation of types restriction.

It also introduces what we might call 'a participatory ambiguity'. The ambiguity consists in the fact that there is a choice that *has* to be made as part and parcel of any semantic assignment. This is a clear violation of the principle of what we will come to call the assumption of 'reasoning transcendent truth'.

6.1. Dunn's ineliminable duality of function violates types restrictions (See Appendix One: i.–iii. and v.)

The perspective index in the robust ternary operators in two-dimensional system is required to fulfill two sorts of role at one time. Thus we get the duality that violates types.[59]

6.2. Dunn's indexing of binary relations cannot condone severally isolated and independent objects (See iii.)

Again the robust perspective index cannot remain discrete — which is to say the indices cannot remain isolated and independent.

6.3. Internal relatedness is expressed in the logic. (i.–ii.)

The ineliminable relatedness is obvious. Consider $\langle 3, 5, 3 \rangle$. The first index represents some world — say the world where all investment bankers are deeply altruistic. The third index, obviously, represents that same world. But back to the first index: It has a second role — it indexes sets of ordered pairs — all those for which world 3 is taken as actual. This cannot be expressed via external relations.

6.4. Indiscretion (iv.)

The internal relatedness manifests indiscretion — which, in turn, is motivated by reasoning transcendent truth.[60]

6.5. Robust versus anemic (i.–v.)

Lindström's system is anemic. That of Rabinowicz and Segerberg is robust. Let's check that out.

7. Rabinowicz-Segerberg vs. Lindström — A case study in robust vs. anemic ternary relations

> Variable-perspective operators are rather peculiar concepts, from the philosophical point of view.[61]
>
> Rabinowicz and Segerberg

F. B. Fitch famously argues that verificationism entails the absurd thesis that: all truths are known.[62] Wlodek Rabinowicz and Krister Segerberg (1994) and Sten Lindström (1997) offer rejoinders. Each reply relies on a two-dimensional logic — but on very different sorts of two-dimensionality. Their distinct approaches exemplify the distinctions between discrete and indiscrete models (and between robust and anemic ternary relations).

Rabinowicz and Segerberg show clearly how an intuitive actualization of the puzzle will collapse[63] for exactly the same reasons that David Lewis first rejected such an approach. Then they go two-dimensional introducing a variable perspective operator — evading Fitch's puzzle. Lindström later tries to solve the problem with a perspective operator — but one that remains fixed within any interpretation. This marks the difference between approaches that might be call 'civilized' (Kaplan; Lindström) and 'monstrous' (Rabinowicz; Segerberg). The details are fairly complicated[64]

— I am going to try to indicate what it is about the two approaches that makes the former a paradigm of discretion and the latter a paradigm of indiscretion.[65]

A first indication of indiscretion comes when Rabinowicz and Segerberg offer the following intuition:

> A formula says something *about* the reference world, but *what* it says is partially determined by the world of perspective.[66]

An external-only restriction on the relations between indices in a model disallows just this sort of interference with the (internal) content of a given point. That is discretion.

7.1. Civilized functions

Lindström's accessibility relations are based on 'civilized' functions:

> For any formula in Lindström's language, its semantical interpretation with respect to a particular model is a function from perspectives to propositional contents.[67]

Within any given interpretation — for example, any indexical constant such as 'this world' will point to an already designated world — whether that be a given reference world (picked out by that interpretation) or a (fixed) perspective world. In order to understand what makes this 'civilized' it might help to see how the Rabinowicz-Segerberg monsters work.

7.2. Monsters

Rabinowicz-Segerberg explain why their account could not be expressed in what I am calling a 'civilized' manner:

> Our epistemic operator, unlike the one he [Lindström] allows, cannot be seen as representing *any* operation on propositions understood as sets of worlds. Nor can it be interpreted in terms of a function from perspectives to such operations.[68]

There are two ways in which this demonstrates important distinctions between the systems of Rabinowicz-Segerberg and Lindström: they rely on different notions of propositions and most importantly the former require indiscretion while Lindström remains discrete.

7.3. Di-Propositions

Philosophical analysis during the Kripke-Lewis-Kaplan reign was expressed almost exclusively in terms of what came to be known as UCLA propositions — sets of pairs of worlds.[69] Models for different logics are *fully* distinguished by various sets of binary relations on pairs of worlds. That amounts to the assumption of discretion — the assumption that the indices or worlds in a modal model can *only* have external relations to one another.

The standard assumption in classical modal logics is that a proposition corresponds to a set of possible worlds. Then the modal operators operate on sets of worlds. This seemingly innocuous formal stipulation blocks indiscretion (such as is found with variable perspective operators).

But there is an alternative. Di-propositions — are sets of ordered pairs of indices.[70] These are utilized in the Rabinowicz-Segerberg system — expanding expressive power.

What is important to note is that to simply *assume* that propositions should be treated in the classical way is to rig the game. It is similar to simple *assuming* that the indices for modal reasoning generally — can be consistent and complete worlds. Another example — simply *assuming* — that the philosophical deck is not stacked by simply assuming that the only sorts of information that we ever need to attend to correspond in structure to set theoretical union.

> It is impossible to view this operation on sets of states as a function from perspectives to operations on sets of worlds.[71]
>
> Rabinowicz and Segerberg

7.4. The Rabinowicz-Segerberg criticisms of Lindström

Lindström utilizes the distinction between *de re* and *de dicto* claims — here with respect to situations. Rabinowicz and Segerberg show that Lindström is unable to keep these categories logically distinct — undercutting any explanatory advantage that distinction might have had for them. The *de dicto* interpretation immediately runs into trivialization. So does the *de re* interpretation *if* the language is to have the power to express such straightforward claims as:

This (situation) is the actual world.

Given such a claim the Lindström verificationist cannot stop a collapse into total knowability (a god-like perspective).

There are a number of further criticisms of Lindström — but even if the *only* one were concerning expressive power — why should a fidelity to reasoning transcendent truth trump that?

> What is distinctive for the present proposal is that we give up the idea of a fixed actual world (the designated point) and replace it with a variable perspective.[72]
>
> Rabinowicz and Segerberg

7.5. Motivation for discretion

At the end of the day what is the motivation for discretion? It may have something to do with a fidelity to truthiness. (See section 9.) But I suspect an even deeper inspiration is a faith in reasoning transcendent truth — the ultimate realist dogma.

8. Anti-Realisms: an application

There has long been a criticism of Michael Dummett's anti-realism that goes like this: He objects to the idealization that is required to explain the grasp of verification-transcendent truth conditions. But Dummett may run into a very similar problem with the grasping of his assertability conditions.

According to the intuitionist a sentence P can be said to be true just in case there is[73] a constructive proof.[74] But there is a notorious open question here. For example, with respect to mathematics:

- Does understanding require an actual proof or *merely* an effectively decidable (possible) proof?[75]

In the former case you seem to be stuck with the restrictions and problems[76] of the strict finitist.[77] But — in the latter case the realization of the conditions for understanding seem to face just the same sort of (idealization) objection, as does the realist account of meaning.

Dubucs and Marion argue that an inert[78] notion of ratification needs to be replaced by a more effective notion of feasibility if the anti-realist is not to be caught by his own objection against realism. Their radical anti-realism does not abide Dummett's notion of grasping assertability

conditions via some method that is *merely* effectively decidable (in principle). Rather the grasp of assertability conditions needs to be feasible in practice.[79] Such feasibility cannot be expressed via an intuitionistic logic. Why?

At this point J. Dubucs and M. Marion jump in with an interesting insight: Intuitionist logic is just as infected with a certain sort of Platonism, as is classical logic.

A Platonist conception of proof as an object[80] sees any proof as just as an assemblage of strings of symbols satisfying certain principles — that is formulas are taken to be types and proofs are taken to be objects (worked upon from the outside). What makes intuitionist proofs Platonist in this sense is the fact that they (like classical logic) embrace the structural principle of contraction — which forces one to treat formulas as types.

For Dubucs and Marion feasibility cannot be expressed via any sort of Hilbert-style calculus — we need to embrace a substructural logic — preferably linear logic (which drops contraction as well as weakening). One characteristic of such a resource-conscious substructural logics is that they can be seen as treating formulas as tokens rather than as types. Another is that a proof is more like an act than an object (some of whose properties the logician simply ratifies). The Platonic notion of proof-as-an-object is satisfied with the notion of decidability-in-principle — an idealized notion to be sure. On the classical view our activity is reduced to that of ratification, which does not require any particular cognitive resources.

There are two prongs to the Dubucs-Marion solution to Dummett's problem:

- A feasibility system (via actual cognitive resources) — which is to be based upon
- A contraction-free logic

One might object to the feasibility approach in that its emphasis on actual cognitive capacity forces a sort of real-world logic that sacrifices the modal generalization we have come to appreciate in our meaning theories.[81] If so one should note that contraction free logics offer *more* than resource consciousness. Contraction free logics open up the possibility for participatory semantics — liberated from the assumption of RTT — the systems can enjoy a genuinely relational semantics such as those that utilize robust

ternary operators or recognition of the possible significance of repetition of 'the same' piece of information.

Clearly one can be contraction-free without embracing the feasibility approach. It is only because Dubucs and Marion assume that they have an epistemic obligation that they are forced to go feasible. Just like with (some) metaphysics — (some) epistemology assumes that we can only act[82] (externally) on a product that is severally isolated and independent.

The assumption that I call 'reasoning transcendent truth' (RTT) is the methodological constraint that would block Dunn's use of *dual* role for the third ('perspective') index in his analysis of ternary operators. The same violation occurs with the robust ternary relations used in two-dimensional analyses of 'actually' in modal contexts. These are examples of the 'monsters' that David Kaplan abhors.

The underlying rationale is that — whatever adjudicates our semantic assignments — must be fixed before any semantic analysis can commence. This is model-theoretic correlate of contraction.

Given such an assumption we are obliged to provide an account of the linkage to that pre-fabricated Platonic reality. But if we define anti-realism via rejection of reasoning transcendent truth — we can avoid such a commitment.

With a contraction free logic we can get not only tokens (instead of types) but also we get for the first time a participatory logic rather than a reasoning transcendent truth type logic. We don't have to *explain* grasping the preformed semantic entities 'from the outside' — we get access to them for free — do to the truly relational nature of the semantics.

The common core of dialectical proof and model theoretic liberations suggests a new cutting of philosophical boundaries — under what (for want of a better name) we might call informationalism. Notoriously classical modal logics generated metaphysical garbage. But there is epistemological damage as well.[83] Once you have contraction free (indiscrete) logic — you no longer have the Platonic baggage which requires metaphysical and epistemic connections. We avoid the struggles over how to account for the practical feasibility. That is because: We are already in the game.

> ... one can clearly identify the structural rules responsible for the vestiges of Platonism that are still present in intuitionism.[84]
>
> Dubucs and Marion

> ... the traditional anti-realist ... pays too much tribute to a «platonician» conception of mathematical inference: according to this conception ... objects may be likened to documents ... already written, perhaps never read by anybody ... [which] could not be affected by limits of the cognitive capacities of the agents.[85]
>
> <div align="right">Jacques Dubucs</div>

> A proof is, not an object, but an act.[86]
>
> <div align="right">Per Martin-Löf</div>

8.1. World theory builds in discretion

In the first class of the first day of graduate school I asked my professor if he could help me choose between two different possible readings of a difficult passage.[87] After staring at the passage for a long time he handed it back to me and said that he could only grasp the proper meaning of any text involving such modal expressions once it had been translated into world theory.[88] 'World Theory exposes the meaning of any philosophical discourse'.[89] Alas world theory guarantees a sanctuary for all the prejudices of the ubiquitous modal philosophical analysis of the latter half of the last century. Chief amongst those is the assumption that models must be discrete and that reasoning transcendent truth cannot be violated.[90]

Restall pointed out:[91] The support between a *point* and a *proposition* need not simply codify information *about points*. It can yield an account of the information carried *by* points.[92]

Not only is world theory not the Rosetta stone for philosophical analysis — it conceals deep prejudices which are manifestly unwarranted and which dramatically restrict expressive power. For example — it condones philosophical pseudo problems. See the reincarnation of essentialism in metaphysics and the epistemic over-reach noted in the rejoinder to Dubucs and Marion.

9. Graham Priest's depraved ternary relations[93]

> What does the ternary relation mean ...?
> It is difficult to give a satisfactory answer to this question.[94]
>
> <div align="right">Graham Priest</div>

Days before finishing this note I received Graham Priest's: Is the Ternary *R* Depraved? It offers a stark contrast with Dunn's take on ternary rela-

tions. First note that even as he used them for relevant B logics in his text on non-classical logic Graham Priest expressed philosophical misgivings about ternary relations. The point of the new article is to address what Priest takes to be the most important objections to the Routley-Meyer semantics for relevant logics.

Just as with modal logic — relevance logic first emerged via proof theory. Many argued that the rebel logic required a model theory to achieve legitimacy.[95] Soon Richard Routley and Robert K. Meyer developed a model theory for relevant logics.[96] Two objections arose — one to the Routley-star semantics for negation and the other to the ternary operator utilized for the conditional. B. J. Copeland[97] led the second attack. Priest is here responding to that attack — by arguing that the Routley-Meyer semantics can be given an informal interpretation that makes them clearly depraved (genuinely semantic).

Priest claims that a logic can be seen as a system that looks at possible patterns of symbols — constrained by certain formation rules.[98] What the logic does is distinguish between those patterns which are valid and those which are not. But Priest insists that a logic cannot be satisfied in just making this distinction. It must also explain *why* certain patterns are worthy and others not. For Priest, an impure or depraved semantics must be given in terms of 'genuinely semantic notions'. Obviously, Priest has clearly not jumped on the emerging inferentialist[99] bandwagon.[100] If depraved semantics are required then inferentialism appears to be dead.[101]

Ternary relations (such as those used for relevant B logics) have frequently been met with skepticism — even at one time by Priest himself.[102]

Priest concedes that we need a ternary operator for the relevant conditional which is expressed via a segregation of worlds into the normal and the non-normal worlds — via a functional relation.[103] So far so good.

But then Priest argues that the semantics needs a little depravity in order to get their explanatory grunt. And where better to find depravity than in the actual world.[104]

The key to achieving his grunt is that Priest defines the notion of 'holding in an interpretation' in terms of a fixed perspective constant.[105]

The original Routley-Meyer semantics did not include a designated world. Priest adds such a world @ which he says represents the situation about which we reason.[106] He claims[107] that the inclusion[108] of a base world in his semantics 'makes no difference to what is valid'. That may

be true for the interpretation that is his quarry — but it certainly is not true for models in general.[109]

One concern is the way that @ constrains possible semantics. The designated @ plays a significant role in delivering depravity — it ties the semantics to the actual world — more importantly it keeps that ground sacred — it cannot permute. Even assuming that that is required for *this* notion of 'justifying' the semantics — it comes at a cost in terms of general expressive power.

An inferentialist obviously is required to justify her explanation of how the meanings of the logical operators are explained proof-theoretically (avoiding Arthur Prior's tonk objection). If such an effort succeeds — Is it not reasonable to assume that there will be a model-theoretic correlate? *If so* it would clearly have to follow Dunn's (and not Priest's) account of ternary relations.

Greg Restall and Graham Priest are clearly two of the finest philosophers on the planet today. The following issue divides them:[110]

> Is it required that the notion of validity be explicitly formulated in terms of the notion of truth?[111]

Consider a recent definition from Restall:

> An inference is valid if it is logically incoherent to assert all the premises and deny all the conclusions.[112]

Notice that this definition does not even implicitly involve the notion of truth. This is Priest's concern. He charges Restall with 'glossing the notion of validity.' Does this rest on something beyond faith in reasoning transcendent truth?[113]

> Validity has got something intrinsically to do with truth.
>
> Graham Priest

> R0 says[114] that, in the situation about which we are reasoning, → really represents the entailment relation.[115]
>
> Graham Priest

> If the ternary relational semantics is to justify the fact that some inferences concerning conditionals are valid and some are not, then there must be some acceptable account of the connection between the meaning of the relation and the truth conditions of conditionals.[116]
>
> Graham Priest

There is not merely a plurality of correct theories and of more or less satisfactory world-views: there is a corresponding plurality of actual worlds ... There is an actual world, which is suitably external, mind-independent, perceiver-independent, and the like, but it is not unique.

<div align="right">Richard Sylvan</div>

Appendix I: Dunn on ternary relations in relevant logics

i. Here is Dunn on the interplay between the static and the dynamic representing the duality between information and computation demonstrating a profoundly relational quality:

> By $R[A]$ let us mean the set of relations determined by the states in A. Suppose we have two such propositions A and B. We can then do various things with $R[A]$ and $R[B]$. For example, we can take $R[A]$ and "*apply*" it to B, getting all the states we can get from B using the relation in $R[A]$, treating A as a *program* and B as *data*. This is the key to modeling combinatory logic.[117]

ii. Again connecting the implicit relational character and duality of function:

> This uses *only* the *implicit* relational character of A.[118] But we can use the *implicit* relational character of A *and* B, taking the relations in $R[A]$ and the relations in $R[B]$ and forming their relative products in all possible ways. This is like viewing *both* A and B as *programs*, and *composing* A with B. This is the key to representing relational algebras. Lyndon showed that relational algebras cannot be represented in the natural way by taking elements to be relations. We in effect show this can be done one type-level higher, by taking elements to be *sets* of relations, i.e., a "relational database."[119]
>
> There is thus a doubling of form. At one time it is reflected into itself; and then it is identical with the content. At another time it is not reflected into itself; and then it is the external existence, which does not at all affect the content.[120]

<div align="right">Hegel 1830</div>

iii. Again on the ineliminable duality:

> One can think of each [indexing] state ρ as having a dual nature, first as a state and second as determining a binary relation $R_\rho \alpha \beta$ between

states α and β. This can be given the "philosophical" reading: "the pair $\langle \alpha, \beta \rangle$ *exemplifies* the relation (determined by) ρ".[121]

Each world in the model can be inert but the indexing world "can be turned in for the set of relations determined" by its index and "those relations can of course be regarded as taking states to states. So A is *at the same time* quite dynamic."

> 'True' and 'false belong among those determinate notions which are held inert and wholly separate essences, one here and one there, each standing fixed and isolated from the other ... [but] truth is not a minted coin that can be given and pocketed ready-made.[122]
>
> <div align="right">Hegel 1807</div>

iv. Indiscretion:

> I wish here to give the main idea. Suppose we have a ternary frame with a set of states U, and a subset A of U. A can be thought of as a set of states, i.e., a proposition. So A is quite static. But, and this is the main idea, it can be turned in for the set of relations determined by those states, and those relations can of course be regarded as taking states to states. So A is *at the same time* quite dynamic.[123]

> What we are dealing with in logic is not a thinking about something which exists independently as a base for our thinking and apart from it.[124]
>
> <div align="right">Hegel 1812</div>

v. Types violation:

> Dunn argues that the indiscrete implicit duality was presaged by John von Neumann's notion of a 'stored program'[125] — breaking free of the prevalent notion of type inviolability.

> The contrast during the first fifty years or so of the twentieth century between the type conscious mainstream in mathematical foundations, and the type insensitive undercurrent in the *logic of computation* is quite striking. It reminds one of the stories of early aviation theorists proving the *impossibility* of powered flight while the Wright brothers and others were working on the first models, and then on *real* airplanes.[126]

Appendix II: Group or Lattice?

An example of indiscretion at work might help. Francesco Paoli's distinction between lattice and group logical operators requires moving beyond an RTT framework.

Consider the following two disjunctions:

[9] Either $2 + 2 = 4$, or London is in Alaska.
[10] Either the butler did it, or the gardener did it.

With [9] no special relationship is assumed to hold between the disjuncts — [9] grounded simply in the acceptance of at least one of the disjuncts [in isolation]. This is a paradigm example of a [Lattice[127] disjunction: II] [Join is associative, commutative and idempotent[128]].

On the other hand [10] has two possible readings:[129]

[L] Assume that I know for sure that the butler is guilty. My sole ground for asserting [10] is that I know one of the disjuncts to be true — thus the disjunction is true 'by logic' — the other disjunct is irrelevant. But this is not the only grounds for asserting [10] though classical logic students might assume so.[130]

[G] Alternatively, suppose that I carried out some investigations and detected that there were only two individuals on the scene of the crime — the butler and the gardener. Say both had equal motive and opportunity — and no one else did. Then I am justified in asserting the following:
   [11] If it was not the butler then it was the gardener and if it was not the gardener then it was the butler.

[11] presupposes a connection between the disjuncts. It is such a connection that produces the acceptance of the disjunction, not the previous acceptance of one or the other disjunct. (Group disjunction or Fission drops idempotence.[131])

The failure of idempotence is connected with the failure of the information theoretic principle of contraction. Note: I need not accept either disjunct in order to be justified in accepting [11] — it is their mutual connection that produces the grounds for the disjunction — not the previous acceptance of one of the disjuncts. Here the disjuncts must be *relevant to each other* — they are not discrete pieces of information. This internal

relationship cannot be expressed if I assume wffs are always type entities — that is if I am bound by contraction.[132]

> Research into an unexplored area like that of the progression of theories in higher order logic, for instance, can't be left to people who expect to have that area mapped out and secured beforehand so they could be advised [*in advance*] as to why they should engage in it.[133]
>
> Uwe Petersen

> Hegel was not a logician.[134]
>
> Timothy Williamson

> In the absence of contraction, normalization and cut elimination can be proved without recourse to the length of the formula in question. It is this that makes logic without contraction so safe against all antinomies arising from abstraction.[135]
>
> Uwe Petersen

> If assumptions are treated as sets instead of multisets, contraction is in a way built into the system and cannot be expressed as a distinct rule.[136]
>
> Jan von Plato

> I was aiming at . . . the possibility of turning [Hegel's'] idea of a dialectic of pure reason to the task of questioning all forms of authority,[137] including that which may be thought to inhere in classical logic itself.[138]
>
> Valerie Kerruish

> Models and possible worlds are extremely different kinds of things.[139]
>
> Patricia Blanchette

> Relations *between* worlds are not sets of worlds.[140]
>
> I. L. Humberstone

> To the ordinary (i.e. the sensuous understanding) consciousness — the objects of which it knows count in their isolation . . . independent . . . resting in themselves.[141]
>
> Hegel 1830

> Giving useful semantics for non-classical logics in the relevance family requires more than just moving from "worlds" to "pieces of information" or "theories." In addition, it will no longer do to have a Kripke binary relation be the only element of semantic structure.[142]
>
> Nuel Belnap

Almost nothing is more commonplace, in the beginning of a book or article on philosophy than to encounter a set of statements about or involving the world ....[143]

Richard Sylvan

Notes

[1] I dedicate this little note to Aida Beatriz Donahue. She was born on July 9, 2009. Congratulations to mom Myrna, dad Sean and sisters — Angie and Carmen.

[2] Wallace (1873), p. 25.

[3] Kerruish and Petersen (2006), p. 81 (I combine parts of two quotes — hopefully faithful to the authors intent).

[4] Kerruish and Petersen (2006), p. 87.

[5] Dunn (2001), p. 9. Dunn is here arguing *on behalf of the fool*.

[6] See Uwe Petersen entries in the bibliography. For the purposes of this note one might begin with his joint paper with Valerie Kerruish (2006).

[7] Obviously there are many central Hegelian theses of Petersen to which I do not attend perhaps most notably the creative role of contradictions in concept formation.

[8] An *uncivilized* interpretation does not respect reasoning transcendent truth — or restriction to discrete information. See section vii.

[9] In many ways this note provides only the skeleton for a much more detailed argument. For example, I claim — without presenting any text from Lewis that his argument against 'actually' enriched logics cannot have certain application, or (more significantly) that Dunn's important work has such and such philosophical implications. Here I do offer text but with far less background explanation than might be desired. And similarly, the important example of the distinction between *robust* and *anemic ternary relations* via the contrast between Rabinowicz-Segerberg and Lindström. In each case I am confident that if the reader goes to the sources my claims will be sustained. But if the full case were to be made this article would be much longer. The point here is simply to note model-theoretic correlates to certain dialectical lessons of a liberated proof theory. The common core can form the basis of a new branch of philosophy which might be called — informationalism. Just the slightest hint in that direction comes from a suggested response to Dubucs and Marion.

[10] This is intentionally kept vague at this point. One could say 'objects of thought' if that notion could be completed drained of any psychologism. We will clarify first model theoretically and then proof theoretically what counts as such objects.

[11] For now you may think of 'objects' as whatever our logic is talking about (of course that is not theory neutral).

[12] External' is, of course, a technical term that has both a proof-theoretic and a model-theoretic reading. We will get to the details soon. For now note that transitivity over worlds would be an external relation. The worlds themselves are like black boxes — any information within them has no effect on the external relations between worlds. The '*monsters*' which David Kaplan warns against are horrific precisely because they cannot be expressed in terms of external relations.

[13] One example of a non-external relations is that between the perspective world (the world temporarily playing the role of actual world) and the other worlds under consideration in deviant two-dimensional operator read as: 'at any world j with j taken as actual'. Another would be required to account for Paoli's group-theoretic disjunction (see Appendix II).

[14] Francesco Paoli is particularly interested in *logical ambiguity*.

[15] Kerruish and Petersen (2006), p. 83.

[16] "The Fregean concept of property is inconsistent with classical logic." J. Myhill

[17] Petersen refers to 'a conflict between form and content'. Dunn refers to an ambiguity that can take one and the same index — *at the same time* as static information and dynamic computation.

[18] The following motivation is almost directly taken from Greg Restall's article on substructural logics in the Stanford Encyclopedia of Philosophy. It is so clear I cannot resist including it. The reader familiar with substructuralism can skip ahead.

What, in general, do we need to know to be justified in inferring β from α?
We need the premise that: *if* α is true then so is β.
That is, we need to know that '$\alpha \to \beta$' is true. In full generality, this is expressed in the deduction theorem.

$$X; \alpha \vdash \beta \text{ if and only if } X \vdash \alpha \to \beta$$

That is, I can validly deduce β from X *taken together* with α if and *only* if I can validly deduce the conditional $\alpha \to \beta$ from X alone.
Note that there are three operational symbols in the theorem: the turnstile '$\vdash$' encoding validity, the '$\to$' for the conditional, and a semicolon ';' encoding the mode of premise combination.
Most standard intro logic texts pay a great deal of attention to the first two and none to the third.
Why is this? *If* we restrict attention to just classical logic then the *left side* of the bi-conditional above could just as well be written

$$X \cup \{\alpha\} \vdash \beta$$

simply *assuming* that the mode of premise combination is that of *set union*. ∪ is the set theoretical operator for union: *idempotent*, *commutative* and *associative*. Each set theoretical principle of union corresponds to a structural rule, for example idempotence corresponds to contraction.

Also: Some ask: Why don't multiple conclusion consequences collapse into one? That is because they have *not* considered the possibility of group disjunction.

[19] This 'neglect' is discussed in footnotes above and below.

[20] This is the structural principle of weakening.

$$\frac{X \vdash A}{X, Y \vdash A}$$

We go *from* a stronger statement, that A follows from X — *to* a possibly weaker one, that A follows from X together with Y. In relevance logics, the inference from an *arbitrary* premise to a *logical truth* such as $q \to q$ may fail. Classical and relevant logics can be expressed with exactly the same operational rules for all of their logical constants. What differentiates them (*all* that differentiates them) is the fact that classical abides by weakening — while relevant logics reject it. One other point — the substructural perspective points out relevance logic does not discover a sieve that needs to be applied to the classical program. It is not just about relevance — irrelevance is a symptom of a more subtle disease.

Relevance has been oversold as a motivating factor in relevance logics.

Meyer and McRobbie (1982), p. 125.

See Stephen Read 2003.

[21] That is — isolated and independent.

[22] Please don't ask me to define 'interesting'. All that I can say is that if your conception of what is interesting is not evolving during your life — neither are you.

[23] Examples of non-classical bunching: Actions: non-commutative fighting and then making up is not the same thing as making up then fighting Syntactic Typing: contraction fails a string of two sentences is not itself a sentence Information Flow: For many channels, you may eke out more information by repeated applications. Any time that using a channel twice (serially) yields more information than using it once will result in a failure of contraction (see explicit proof content). Paradoxes generated by *pseudo modus ponens* are vanquished by rejecting contraction.

[24] This generates a devastating argument *against* classical monism:

You mean to say that you insist that the *only* form of information bunching that we *are allowed to consider in logic* — our most *general*

discipline — is that which corresponds to *set theoretical union*? That is highly restrictive — and what is worse — it is arbitrary.

[25] This lovely phrase comes from John Slaney (1990).

[26] Intuitionists drop contraction right — which is responsible for LEM.

[27] Consider a judge drawing an inference from three instances of the same type of info say identical eye-witness accounts as against just one.

[28] Obviously there are many more.

The distinctive feature of contraction-free fuzzy logics is prelinearity — which can demonstrate the cost of running a program.

<div align="right">Libor Běhounek and Petr Cintula</div>

Contraction is the reason for the undecidability of first-order logic. If contraction is excluded, then there are no infinite paths in the proof search and thus derivability becomes decidable.

<div align="right">Andrea Cantini</div>

There is a way of organizing the principles of proof so that one can start from the theorem to be proved, then analyze it into simpler parts in a *guided way*.

<div align="right">Sara Negri and Jan von Plato</div>

[29] Following an early insight by the Russian logician V. N. Grišin, Uwe Petersen argues that contraction is the only rule in which the induction on the length of a cut formula is required without exception.

[30] As Restall notes: For many channels, you may eke out more information by repeated applications. Any time that using a channel twice (serially) yields more information than using it once will result in a failure of contraction.

[31] Jacques Dubucs and Mathieu Marion (2003) and Jacques Dubucs (2002).

[32] There is a more subtle advantage — yet to be revealed.

[33] Dubucs (2002), p. 232.

[34] For now think of discretion as the assumption that worlds can have *only* external relations (like transitivity) with respect to one another. With respect to information they are like black boxes. Monsters violate this constraint.

[35] The information at a state here is its data — in world-theoretic terms — what is true there. Dunn's whole point is that this leaves out a critical relational element — which is formally expressed via partial functions.

[36] See Dunn's quote at the head of this note.

[37] Not a lot of philosophers have appreciated the commonality between relevant and 'actually' enriched logics. Richard Sylvan (1997) is a notable exception.

[38] Hughes and Cresswell (1968).

[39] Abstracting away from the particulars *any* indexed logic requires: points or indices that support information. Furthermore, the information carried by

a point must have some sort of logical coherence. Systems that exemplify such indexed logics include:

- Intuitionistic and modal logics
- Relevance and Linear logics
- Maps

The received view in philosophy — until very recently — was that standard modal logics provide the paradigm case of an indexed logic.

[40] The conditional of a relevant B logic exploits the ternary relation $Rxyz$ — which can be read as saying — if we start with the perspective of state x then if we were to enter state y then state z is a potential outcome. (That is, the pair $\langle y, z \rangle$ is a possible transition given state x.) This allows us to place restrictions on this rule based on the base or normal world that can block the undesirable inferences. Basically the third index quarantines the non-normal worlds so that the advantages of relevance can be established while we build up a stronger logic than N*. This is a very *different* role than the third index plays in (fully) actualized modal logics. But in both cases we end up with situated models that cannot be expressed with discretion.

[41] This 'challenges' rather than 'obliterates' discretion because it is possible to neuter the challenge by requiring that the ternary operators be *anemic* — as with 'the war on monsters'.

[42] External relations are forced by any discrete semantics. The first quote from Dunn at the head of the paper is mocking such discretion. Model theoretically internal relatedness (truly relational semantics) will be accommodated by the double-functioning of certain indices — violating types restrictions.

[43] It is exactly this that violates *Åqvist's constraint* and David Kaplan's Constraint (against *monsters*) and which Dunn is celebrating. According to *Åqvist's constraint*: if two points differ *only with respect* to their (second) perspective coordinate then they must agree on their assignments to atomic statements.

[44] Philosophical nonsense is a further cost.

[45] There is an obvious need for an actuality operator in our modal systems — for consider

It is possible for everything which is in fact φ to be ψ.

$$\Diamond (\forall x)(\mathsf{A}\varphi x \rightarrow \psi x)$$

This *cannot be expressed* without an actuality operator — A. The first somewhat natural expression in a formal context was to have a *distinguished world* amongst the possible worlds representing our beloved (contingently) actual world. Then the actuality operator can always point to that world. This is a bad idea for at least two reasons:

- This will guarantee as a thesis: $A\theta \rightarrow \Box A\theta$. This is unacceptable *as a thesis* because from the simple fact that world α is actual it ought not to follow that no other world *could* have been actual.
- Any *actualized* theoretical principle (say a metaphysical thesis) faces a dilemma:
  Either the principle will be *ethnocentric* (attributing some special metaphysical status to our contingently actual world) *or* the *expressive advantages of actualization will be sacrificed*. Indeed this is exactly the dilemma that David Lewis threw in the face of any attempt at actualizing a philosophical principle.

What is needed is a two-dimensional modal logic that has strong enough power to express: *Whichever world had been actual* — α would have been true at *that* world *considered as actual*. So, for example, $A\alpha \rightarrow \Box A\alpha$ would not be valid with the modality in its consequent read '*at any world with that world taken as actual*'. We introduce a *new* necessity operator Φ which simultaneously does two things: it ranges over *all* possible worlds and on each instantiation it sets up *a marker* to which each subsequent actuality operator (in a given wff) will point. Thus, $A\theta \rightarrow \Phi A\theta$ is not a thesis.

[46] Imagine you have a generalized modal principle which is subject to a killer objection. (Perhaps we cannot get a certain argument of which it is a constituent valid — when, intuitively, it clearly seems to be valid.) It may seem reasonable to add an actuality operator within the syntactic scope of the wide scope necessity operator that provides the generalization — perhaps because this gives you your validity. You have just actualized the principle. See the tolerance principle.

After all as long as we remain one-dimensional there are only two scopes: $\Box \ldots \downarrow A \ldots$ *or* $\downarrow \Box \ldots A \ldots$.

[47] If you read the actuality operator as inside the scope of the widest modal operator it is impotent — but if it is scoped outside the necessity operator we get ethnocentrism. The key point is that with a one-dimensional language those are the *only* options.

[48] Lewis himself gets around this expressive inadequacy with his indexical analysis of 'actually' as part of his later pluralism. An objection to that approach is beyond the scope of this note. I would argue that (perhaps surprisingly) that his pluralism itself relies on the assumption of reason transcendent truth. Once that fix is dismissed the obvious alternative is a two-dimensional logic.

[49] This expression is ripped off from Hintikka's arguments about quantifiers: two completely different issues. Or are they?

[50] It is clearly follows from Lewis' argument in the applied case.

[51] This principle of tolerance — from the Four Worlds Paradox — is one requiring generalization.

[T] Any artifact could have been constituted out of a collection of parts slightly different from the collection out of which it was *actually* constituted.

My bicycle could have been made out of a collection of parts slightly different than the collection out of which it was *actually* made.

[52] The most general feature is the assumption of reasoning transcendent truth.

[53] For example, Consider what came to be known as UCLA propositions — sets of pairs of worlds. Models for different logics are *exclusively* distinguished by binary relations on pairs of worlds. That amounts to an expression of discretion — the assumption that the indices or worlds in a modal model can only have external relations to one another. An expressive advantage occurs if we allow di-propositions (See Segerberg and Humberstone).

[54] Of course there are a number of more familiar objections to Lewis' pluralism. I am working on a paper which will lay out the realist objection.

[55] At this point I direct the reader to two articles. Lindström (1997) exemplifies the anemic use of two-dimensions. Rabinowicz and Segerberg (1994) exemplifies the robust use of two-dimensions. I will not review the formal details of their analyses. Rather I point out features of those analyses sufficient to disclose the issue of discretion.

[56] Dunn celebrates the type violation with respect to ternary operations for relevant logics — but the same expressive advantage is enjoyed here — perhaps with even more intuitive support.

[57] Contrast the original Åqvist and Segerberg 1972 articles.

[58] Routley (1979), p. 302.

[59] See Appendix II Paoli on Group-theoretic disjunction.

[60] I originally referred to this as 'reason transcendent truth'. My son pointed out that this has a prejudicial interpretation (which I enjoyed). But I now defer to his sense of legal fairness.

[61] Rabinowicz and Segerberg (1994), p. 108.

[62] Fitch (1963), Rabinowicz and Segerberg (1994), Lindström (1997).

[63] Rabinowicz and Segerberg's key point with respect to the Fitch argument is that: There are frames that satisfy their semantics and in which truth still does not imply knowledge: That is they allow propositions that contain some self-centered states without containing all their epistemic alternatives. As a result, in some models on such frames, for some formulas φ and some self-centered states, $\varphi \,\&\, \neg \mathsf{K}\varphi$ is true in those states.

[64] Instead of evaluating a formula at a single point (world) each formula is evaluated at *an ordered pair* of points intuitively consisting of a perspective world and a reference world. The role of the *designated world* is taken over by the *set of* self-centered states — those of the form $\langle w, w \rangle$. They offer a

simple proof that in their new system $\varphi \to \mathsf{KA}\varphi$ is invalid! (106). Thus, Fitch's argument is 'actually' buried.

[65] One way they differ is that the monsters violate Åqvist's constraint. That is at least a sufficient condition for escaping civilization. Rabinowicz and Segerberg actually allude to this difference — concluding (incorrectly in my view) that it is not important. Part of the reason they may have concluded this is that they are focused on different criteria of what I am calling being uncivilized.

By the way, for the reader who is surprised to hear 'civilized' being used in a pejorative way — I suggest that you take a closer look to history. The self-proclaimed 'civilized' ones are always the most horrific.

[66] Rabinowicz and Segerberg (1994), p. 104.

[67] Rabinowicz and Segerberg (1994), p. 109. This is important. Rabinowicz and Segerberg cannot be interpreted as a function from perspectives to propositional contents. They need more expressive power than that can offer. This is relevant to the discussions of di-propositions and of Priest (below).

[68] Rabinowicz and Segerberg (1994), p. 110.

[69] This corresponds to what Dunn calls the static notion of a proposition. Note that in the Priest article mentioned below — he embraces the classical notion of a proposition.

[70] The first use of di-propositions that I am aware of appeared in Humberstone (1981). One might call these 'hyperintensional'.

[71] Rabinowicz and Segerberg (1994), p. 110.

[72] Rabinowicz and Segerberg (1994), p. 101.

[73] There are two interpretations of 'is' here. According to one there must be an *effective procedure for constructing such a proof*. According to the other we must be *in actual possession of such a proof*.

[74] Constructive proof is usually defined inductively. This would include, for example, a constructive proof of an existential sentence $\exists x P(x)$ via an *effective procedure* that gives us a certain object a and a proof that $P(a)$ holds.

[75] Dummett realized he faced such a question when he wrote *The Elements of Intuitionism*.

[76] For an example of objection *to* strict finitism see Mitchell (1992), p. 441:

> An adequate theory of the practical abilities we actually possess requires that we be accorded possession of practical abilities we cannot actually exercise.

[77] Dubucs and Marion (2003) explicitly acknowledge *strict finitism* as an inspiration for their approach. An example of what generally is thought to separate the strict finitist from the intuitionist would be a sentence that is decidable according to the intuitionist but which includes numbers that are too

big to decide in practice. To be fair *feasibility* is strictly an alternative to strict finitism for these authors. But for our purposes — they are more like cousins — they share many traits — none of which we wish to embrace.

[78] In part 'inert' means that proofs are treated as objects.

[79] Dubucs and Marion (2003) eschew psychologism — but their approach is a little overly epistemic for my tastes. I hope to account for the participatory ambiguity of a liberated logic in purely information-theoretic terms.

[80] Would it be too outrageous to at least open a discussion on whether the commodification of absolutely everything in contemporary society is somehow connected to RTT? Treating proofs a merely objects is in a way an abdication of responsibility with respect to the proof-theoretic environment.

[81] Some objections to strict finitism could also arise.

[82] 'Act' may be a misleading expression here. Dubucs and Marion (2003) reject any reference to explicit cognitive capacities in their theory of feasibility. The coherence of their dependence on implicit cognitive capacities is worrisome — but beyond the scope of this note. See Marion (2008).

[83] We ought not assume that the boundaries of what we count as metaphysics, epistemology, or certainly logic are set. If I am right substructural insights ought to give us pause to consider the boundaries of epistemology. The lack of a fix methodology (and even subject matter) is what makes philosophy so interesting. See the last quote in this note: Graham Priest: The nature of philosophy and its place in the university, Inaugural Address University of Queensland, 1989.

[84] Dubucs and Marion (2003), p. 246.

[85] Dubucs (2002), p. 226.

[86] Cited after Dubucs and Marion (2003), p. 241.

[87] It was a part of the Dummett-Kripke debate.

[88] World theory is the LPC theory that translates modal operators into quantifiers over worlds.

[89] It sounded like we had found the philosophical Rosetta Stone. More seriously, if standard modal logics are inadequate — because of not being relevant or not (at least) paraconsistent or for some of the even more interesting reasons implied by this note — then it stands to reason that LPC is somehow inadequate (especially with respect to expressive power) as well. We know that there are a number of (more expressive) alternatives to LPC. It would be interesting to see which, if any, quantified language could express — say linear logic. What about Hintikka's IF logic or Paoli's group-theoretic quantifiers?

[90] Another professor held a hammer over my head while screaming 'I will not allow you to violate Åqvist's constraint'. I reminded him how difficult it would be to explain Åqvist's constraint to the Amherst police — although I did concede that once they did grasp it they would rule — justifiable homicide. (Of

course it was not a real threat — but you have got to have your fun while you can.)

[91] In his PhD dissertation.

[92] Hear this insight from Greg Restall concerning relevant models:

The support between a *point* and a *proposition* need not simply codify information *about points*. It can yield an account of the information carried by points . . . .

The crux of the matter is the notion of "support" between a point and a proposition. Does this relation simply codify information *about* points in a model, or does it somehow give us an account of the information *carried by* points? The former is an external perspective, as we are classifying points from "the outside". The support relation is a way of classifying points. The latter account is the *internal* perspective . . . .

Modal logics are [taken by the external perspective to be] . . . useful sublogics or *refinements of first-order claims about models*. The standard translation ST of a formula is taken very seriously indeed. Intensional formulae are codifications of what is true *about* points. [See the section on world theory.]

If you take an *external perspective* on a frame Boolean negation is warranted.

If you take an *internal perspective* it is quite likely that Boolean negation *cannot even be defined*. According to the external perspective [the consequence relation] codifies information about points in a structure.

[93] It is interesting that while many of the leading lights of the non-classical revolution (Stephen Read, Uwe Petersen, Greg Restall and Francesco Paoli) have come to embrace inferentialism — Graham Priest is not even slightly tempted.

[94] This quote appears in both the 1st and 2nd editions of Priest's *Introduction to Non-Classical Logic*. He addresses the issue in detail in his (2009). His new views on ternary relations are very different from Dunn's. I say a few words about them below. I am willing to bet that a case can be made that his sort of ternary relations cannot match the expressive power of the Dunn-Rabinowicz-Segerberg ternary relations (although I do not make that case here). Assuming that I did make that case it is open to Priest (or other model-theoretic conservatives) to reply: 'So what? The connection of models to truth is more important than expressive power.' That would set the table for a truly interesting battle.

[95] Etchemendy (1990) notes that although both soundness and completeness proofs are required for any logic model theory is generally thought to justify what proof theory delivers. Etchemendy's attack on the standard Tarskian model theoretic notion of logical consequence is not irrelevant to the question at hand. Patricia Blanchette supplements that argument in her (2000).

There is a fascinating philosophical standoff between Graham Priest and John Etchemendy that at least ought to be mentioned here. See Etchemendy (2008).

[96] Routley and Meyer (1973).

[97] Copeland (1983).

[98] The formation rules pick out the well-formed-formulas — and let's assume (without loss of generality) that we are dealing with patterns that have at least one premise and exactly one conclusion. The latter assumption is not lame — but it does not matter here.

[99] For Priest's argument against inferentialism see his (2006b), Chapter **11**. At one point Priest (almost) says that intrinsic semantics must be grounded in the notions of sense and reference.

[100] Members of the band include Greg Restall and Francesco Paoli. I am more than sympathetic to inferentialism — but one point of this note is to argue that it is wrong to assume no model theory can be copasetic with this inferentialism.

[101] Why 'appears'? Maybe it is dead only if it assumes explanatory superiority over any model-theoretic approach — that is if indiscretion is allowed. But clearly on Priest's restricted notion of what count, as a ternary operator inferentialism cannot even be sufficient.

[102] Graham Priest's Introduction to Non-Classical Logic 10.6.

[103] This allows one to build on the system N* which is absolutely free of irrelevance but which has been left very weakened.

[104] Can I hear 'Amen'?

[105] Similarly, a key to Lindström's avoiding monsters is his decision to rely on rigid situation constants.

[106] But see the quote of his revolutionary buddy Richard Sylvan (né: Routley) at the end of this note.

[107] Footnote 20 on p. 10 of Priest (2009).

[108] Graham Priest notes it does *not* appear in the original Routley, Plumwood, and Meyer and Brady semantics.

[109] See for example the difference between 'general' and 'real world' validity in Humberstone and Davies: Two Notions of Necessity. Also note Rabinowicz and Segerberg's notion of weak validity in their (1994) is similar to sort of real-world validity favored by Graham Priest.

[110] Restall (2005), (2008), and Priest (2008b).

[111] Not however *truth simpliciter* — since Priest holds both true and false as a distinguished value — to be preserved by any valid argument.

[112] Restall (2008).

[113] Here is an irony. The criticism by the RTT folks is that the alternative is 'merely formal'. But it is the old way that guarantees the petrification of concepts.

[114] R0 simply stipulates that @ is a member of the set of normal worlds. What this achieves is fixing the third parameter for Priest guaranteeing a respectable but anemic notion of validity.

[115] Priest (2009), p. 13.

[116] Notice the model theoretic bias — again.

[117] Dunn (2003), my emphasis.

[118] Better — uses the implicit relational character of only A.

[119] Dunn (2003), my emphasis. Ternary relations' graphs — just an equilateral triangle with one side an arrow (pointing to, say, the last position).

[120] Wallace (1873), p. 189.

[121] Dunn (2001), p. 8.

[122] Miller (1977), p. 22.

[123] Dunn (2001), pp. 10 f.

[124] Miller (1969), p. 50.

[125] Arthur W. Burks, Hermann Goldstine and John von Neumann: Preliminary Discussion of the Logical Design of an Electronic Computing Instrument, 1963.

[126] Dunn (2001), p. 5, my emphasis.

[127] It is important to note that lattice disjunction is not the same as the inclusive disjunction provided by the truth tables. (See Paoli (2007) and (2008).)

> Truth tables are bad meaning-assignment devices they may give rise to dangerous equivocations.
>
> Francesco Paoli

[128] Here is why it is idempotent: A II A is accepted in virtue of the acceptance of one of its disjuncts if and only if A is *itself* accepted.

[129] These examples and the commentary are taken directly from Paoli (2007) and (2008).

[130] Obviously knowing the other disjunct to be true would do just as well.

[131] Here is an indication of the failure of idempotence: I can accept the first disjunct of [10] *without* accepting:

[12] Either the butler did it, or the butler did it.

with an 'intensional' "or" — if I accept the possibility of the alternative possibility that the culprit was the gardener.

[132] By 2008 Paoli recognizes a third superintensional disjunction.

[12] Either it will rain, or the match will be played.

With [12] a tacit *ceteris paribus* clause is attached to the first disjunct awarding it a privileged role. Superintensional disjunction: loses both commutativity and idempotence.

[133] Petersen (2002), p. 963.

[134] While conceding, of course, that Hegel did not enjoy access to the advances in formal logic that we do — I have to agree with Petersen that this

makes Hegel's logical intuitions all the more impressive. Those intuitions have anticipated some of the most subtle philosophical corollaries of the substructural revolution in logic both proof-theoretically and meaning-theoretically. The quote is taken from an interview in a magazine titled '3 AM'. Of course my comments are not intended as a criticism of Williamson. No one can be expected to be coherent at 3 AM.

[135] Petersen (2000), p. 374.

[136] Sometimes it is instructive to operate at less than the most general level. We have noted that set union (if the classical logician were right — the only way that premises in an argument or items in data bases can be bunched together) is *associative*, *commutative* and *idempotent*. Sequents allow maximum possible variation from sets. On the other hand multisets *union* is like set union in being associative and commutative but multiset union does not follow set union in being idempotent.

The most basic structural rule on sets is extensionality — saying that *inference from a set need only appeal to a subset*. There need be no such constraint on *multisets*.

While extensional conjunction follows from the set of its conjuncts and in turn entails each of them: "Fusion [non-classical conjunction] is entailed by the multiset of its conjuncts and in general entails not those conjuncts individually but whatever can be got by using one of them as an inference ticket and the other as a departure point." (Slaney (1992), p. 315.)

[137] Reasoning Transcendent Truth provides a great deal of such authority. This topic has political implications.

[138] Kerruish (2006), p. 25.

[139] Blanchette (2000), p. 67.

[140] Humberstone (1978).

[141] Cf. Kerruish and Petersen (2006), p. 80.

[142] Belnap (2003), p. 174 f.

[143] Sylvan (1997).

Suppose that an inquiry is self-reflexive. Then *prior* to the inquiry we have no independent fix on the nature of its object. But if we have no such fix, we have no fix either on the fundamental assumptions of the special methods of the inquiry.

Graham Priest

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